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Structures in Two-Choice Reaction-Time Data

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Multidimensional scaling (MDS) analyses were carried out on reaction-time data obtained in *same-different* judgments between pairs of stimuli from various stimulus sets: Munsell colors varying in chroma and value, circles varying in size and radius inclination, parallelograms of varying size and tilt, and rectangles varying in height and width. Three main purposes were served by this study: (a) to illustrate a maximum likelihood estimation procedure for metric MDS applied to speeded *same-different* judgments, (b) to compare the spatial representations of various stimulus sets derived from dissimilarity data obtained in conditions of data- and resource-limitations with the spatial representations of the same stimulus sets derived from dissimilarity measures collected in conditions of unlimited viewing and/or responding, and (c) to examine the concepts of integrality and separability as they apply to the dimensional decomposability of stimuli. The results showed that although the same spatial metrics best represented perceived dissimilarities for each subject within each stimulus set, individual differences and divergence with results obtained on similar stimulus sets in different conditions were apparent. Some implications of these results for research on perception of multidimensional stimuli are suggested.

One of the longest standing problems in perception has been to uncover the transformation that maps objective variations in a stimulus onto a "psychological" scale that is an invariant function of physical intervals. This was a question addressed by the first "scientific" psychologists in Germany, and researchers in unidimensional psychophysics have since devoted much work to determining the function relating the physical scale to the psychological scale (e.g., Stevens, 1951). Findings in this field of research have shown that the perception of differences between several instances of a class of stimulus is not a simple invariant function of their physical variations. In comparison, it is only recently that the same question has been addressed with respect to stimuli that vary along two or more dimensions, and the complexity of the problem has become considerably larger. Not only two or more physical dimensions have to be mapped onto a psychological scale, but variations along one dimension are not without effect on how another dimension is perceived—a situation that requires the identification of the rules governing how two or more dimensions combine psychologically in order to determine the perception of multidimensional stimuli. One of the current approaches to examining such combination rules is the study of similarity relations among a set of stimuli, and these relations have generally been treated as distances in a representational space. Multidimensional scaling (MDS) analysis provides one means of examining a limited class of combination rules, and it is the purpose of this study to carry out such an examination, using as similarity measures reaction-time (RT) data obtained in *same-different* judgments between pairs of stimuli from various stimulus sets.

Perceiving multidimensional stimuli requires complex interactions between a stimulus and an observer. These interactions involve the nature of the stimulus dimensions and the perceptual capacities of the processing organism, and they are influenced by the relative separations between stimuli in the discrimination space and by the particular conditions and requirements prevailing in the experimental setting.

Multidimensional stimuli have properties that vary with the nature of their dimensions. Some stimuli can readily be analyzed into their component dimensions, whereas others are perceived as one complex dimension that encompasses inherent subdimensions not easily seperated from one another. Stimuli varying along the dimensions of shape and color (Handel & Imai, 1972), of circle size and radius inclination (Hyman & Well, 1968; Shepard, 1964), and size and tilt of parallelograms (Attneave, 1950; Hyman & Well, 1968) belong to the former category in that their component dimensions are readily separable and perceptually distinct; that is, one dimension does not interfere with the perception of the other. Their representation in a dimensional space is appropriately described by a city-block metric. On the contrary, stimuli of constant hue varying along brightness and saturation (Torgerson, 1958; Hyman & Well, 1968) or stimuli of convergent oblique lines varying in angle and length (Smith & Baron, 1981) belong to the latter category because they are perceptually homogeneous as if their component dimensions combined into one complex dimension. They are compared with each other on the basis of their global similarity instead of their component dimensions and are best represented by the Euclidean metric in a dimensional space. Thus, the distance of a set of multidimensional stimuli varies with the nature of the dimensions: A city-block metric

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serves well the spatial representation of analyzable stimuli, to the dimensions of which subjects attend selectively, with the perceived distances summed along each dimension; the Euclidean metric provides a more appropriate approximation of the spatial representation of unanalyzable stimuli, and subjects seem to make judgments of perceived distance directly on the basis of their overall dissimilarity. Some recent criticism of these views will be examined in the Discussion.

Although the *stimulus* side of the interaction represents an important factor in determining how the organism processes information (e.g., Garner, 1970), the particular capacities of the *processing* organism to operate along the various dimensions of a stimulus must also be examined. There is, for example, evidence that stimuli that are analyzable by normal adults are perceived as integral and unanalyzable by children (Shepp, 1978), suggesting that the particular properties of the stimulus cannot by themselves account for the way specific multidimensional stimuli are perceived. Repeated exposure to stimuli may allow for component dimensions to "stick out" more distinctly (e.g., Pick, 1965), and an initial failure to attend selectively to a relevant dimension, as indicated by orthogonal interference, may be overcome with practice (e.g., Dykes, 1979). There is also evidence of individual differences in the capacity to implement "integral" and "separable" types of processing. Although the basis for these individual differences remains unspecified, there are some indications that in adults the subject's cognitive style influences whether stimuli are perceived along their global similarity or their component dimensions (Smith & Baron, 1981). The characteristics of integrality and separability do not constitute, therefore, static properties of a stimulus set or of a processing organism, but are the end product of dynamic transactions between an individual and the environment.

Because stimulus and processor interact to determine how multidimensional stimuli are perceived, suggesting that properties of a stimulus are relevant only with reference to an observer, two factors influencing each side of the interaction must also be taken into consideration when investigating the nature of the processes involved: (a) the particular characteristics of the stimulus as they can be extracted by the sensory system of the perceiver and (b) the particular performance conditions imposed on the observer.

A large number of studies on visual perception have been carried out through tachistoscopic presentation in which information is presented for a brief duration, which affects the stimulus contents that can be reliably extracted and subsequently processed by the perceiver. Stanovitch (1979; see also Santee & Egeth, 1982), for example, has suggested that in such conditions, processing may rely on partial information from the stimulus, which may affect how dimensions are combined, in comparison with unlimited viewing conditions. Lockhead (1972, 1979) has suggested that a stimulus is first perceived and processed "integrally," and research on the microgenesis of perception (e.g., Flavell & Draguns, 1957) indicates that during the first milliseconds of exposure only the general configuration may be reliably extracted. Thus, the mode of presentation may determine the stimulus characteristics that are the most salient and available for processing, and it influences the particular nature of the dimensions as they are perceived by an observer. Thus, stimulus dimensions that are separable under unlimited viewing conditions may not be as distinct perceptually when presented very briefly.

The time allotted to the subject to produce a response constitutes an additional factor contributing to the particular processes that can be implemented. In conditions that use speed as dependent variable, the respective salience of the dimensions plays a critical role in determining how the dimensions are processed (Sergent, 1984), and differences in processing stimulus dimensions may emerge, depending on whether speed requirements are involved. For example, Smith and Baron (1981) observed that the amount of interference produced by the irrelevant dimension in the classification of integral stimuli was significantly different and uncorrelated in timed and untimed tasks. Similarly, Takane and Sergent (1983) found the eyes as the most salient dimension in a set of face stimuli in an untimed dissimilarity judgment task, whereas the same subjects comparing the same faces in a two-choice reaction-time task gave less weight to the eyes than to the hair or the jaw in their dissimilarity judgment.

Thus, data limitations inherent in tachistoscopic presentations and resource limitations inherent in reaction-time *same–different* judgments (Norman & Bobrow, 1975) introduce conditions that directly influence the characteristics of the stimulus that can be reliably extracted and the particular processes that are implemented. A large body of research on visual perception and processing of multidimensional stimuli has been carried out under these conditions, and reaction time has proved to be a reliable measure of confusability, allowing inferences about similarity relations (e.g., Monahan & Lockhead, 1977; Podgorny & Garner, 1979). On the other hand, research on the integral–separable dimensional properties of stimulus sets has essentially relied on data that were collected in experimental settings involving no such limitations—as in free classification, rank ordering of similarity, direct distance scaling, dissimilarity rating—which may all require processes unlike those underlying speeded *same–different* judgments. The development of a new MDS procedure (Takane & Sergent, 1983) designed to analyze reaction times obtained in the context of *same–different* judgments may then provide the possibility to study the transformation from physical to psychological scaling of multidimensional stimuli, as well as the underlying combination rules, under conditions typically used in visual perceptual research.

Experiments

Four independent two-choice reaction-time experiments were conducted on 2 subjects. The sets of stimuli presented were similar to those used in previous MDS studies based on dissimilarity measures other than reaction time, and they comprise Munsell colors of constant hue varying on value and chroma, circles of different size containing a radius of different inclinations, parallelograms of varying size and tilt, and rectangles varying in height and width.

Models

The basic principles underlying the joint analysis of reaction times and *same–different* judgments will be briefly outlined in nontechnical terms. A detailed, and more technical, description and justification of the model has been presented recently (Takane & Sergent, 1983).

Following the tradition of multidimensional scaling, we describe the stimuli as points in a multidimensional space in such a way that the interpoint distances best represent the similarity relations between the stimuli. The stimulus dissimilarities are then assumed to be related in a specific way to the observed reaction times and *same–different* judgments. However, because of random error perturbations in the judgmental process, this relation is stated only in probabilistic terms. Thus, there are three important ingredients that comprise the model (Takane, 1981): what distance function is used to capture the stimulus similarity (the representation model), how random errors enter into the judgmental process (the error model), and how the error-perturbed distances are transformed into specific forms of observed data (the response model).

The *representation* model used in the present analysis belongs to the class of distance functions called *Minkowski power metric models,* with power being either 1 (city-block space) or 2 (Euclidean space). The *error* model specifies the nature of error perturbations operating on the distances. A lognormal distribution was assumed in the present study, as suggested by Takane and Sergent's (1983) analysis of the distribution of the residuals. The error-perturbed distances are further related to the specific form of the observed data—reaction time and *same–different* judgment in the present study—by a specific *response* model. Using the rationale of signal-detection theory, we assumed that a threshold value discriminated the error-perturbed distances into *same* and *different* judgments. Reaction time, on the other hand, we assumed to be a negative power function of the absolute difference between the threshold and the distance separating the stimuli (both the exponent and the intercept are allowed to be free in this model). The rationale is that if the difference is small, the judgment is more difficult and consequently it takes more time. The three basic models are combined into a coherent model that relates the interpoint distances to the observed distributions of reaction times and the probabilities of *same–different* judgments. The maximum likelihood estimation is used to determine the values of model parameters (stimulus coordinates in the distance model, etc.) in such a way that the likelihood of observed data is a maximum. See the General Method section for an advantage of maximum likelihood estimation in the context of the present study.

For each experiment and subject, the best approximation of the data was sought under two different conditions: One consisted of the usual multidimensional scaling analysis in which dimensionwise differences between stimuli are summed across dimensions to define overall distances that are related to observed dissimilarities through the model described earlier. Dimensions are derived entirely on the basis of the observed dissimilarities and are not in any way constrained by extraneous information about the stimuli. Thus, by contrast with most reaction-time experiments in which relevant dimensions are defined a priori and without validation (e.g., one assumes that ellipses are defined by their height and width, not by their shape and area; see Dixon & Just, 1978), MDS analysis provides a solution that is not influenced by any a priori specification of the relevant dimensions. Consequently, it is called the *unconstrained* solution. On the other hand, on the basis of knowledge of the manipulated dimensions of a stimulus set, it is possible to impose a series of restrictions on the representation model. For example, the Munsell Company provides its color patches with identified dimensions of value and chroma between each sample belonging to a particular hue. If, indeed, those are the dimensions defining psychological distances between the colors, then stimuli having equal value or chroma according to the Munsell system should have equal coordinate values on the respective dimensions defined by MDS. That is, one can "constrain" the model to fit the data under this hypothesis and examine whether the obtained solution provides a better approximation than an unconstrained solution. If the *constrained* solution represents the best solution, one may conclude that the psychological distances are represented fairly well by the prescribed dimensions in the stimulus set. If the unconstrained solution is the best, one may conclude that the psychological distances between the stimuli are not well represented by the prescribed dimensions, and further examination of the derived configuration is required.

Several specific objectives were served by these experiments. The first was to validate a maximum likelihood estimation procedure for metric MDS applied to speeded *same–different* judgments. The theoretical foundations and the mathematical development of this procedure have been recently presented, along with an assessment of its reliability through an analysis of data from a face discrimination experiment (Takane & Sergent, 1983). The use of different sets of stimuli in this study was intended to examine the versatility of the model and to test its validity with multidimensional stimuli whose spatial dimensional representation has already been investigated through different procedures.

Another objective of this study was the comparison of the spatial representations of the various sets of stimuli derived from dissimilarity data obtained in conditions of data- and resource-limitations with the spatial representations of these same sets of stimuli derived from dissimilarity measures collected in conditions of unlimited viewing and responding. Specifically, will judgments on the color stimuli conform to the two-dimensional Euclidean model, and will judgments of the geometric stimuli whose dimensions are distinct conform to the two-dimensional city-block model? In addition, the present study was concerned with determining the goodness of fit of the derived spatial representation of the stimuli with the configuration of these stimuli derived from the objective variations between each member of the set. Specifically, are the psychological dimensions defining dissimilarity between stimuli of a set one-to-one mappings of prescribed dimensions of these stimuli? These questions will be answered by model comparisons in this article. The specific model-comparison procedure will be described at the end of the General Method section.

This study was essentially exploratory in nature, and two factors of importance in the evaluation of similarity relations among stimuli were not considered. First, the power exponent of the Minkowski distance metric was restricted to either 1 or 2, corresponding to the city-block or the Euclidean distance model, respectively, even though neither value may actually provide the best fit of the data. For example, Handel and Imai (1972) and Hyman and Well (1968) showed that the best MDS solution was often obtained with an exponent between 1 and 2,

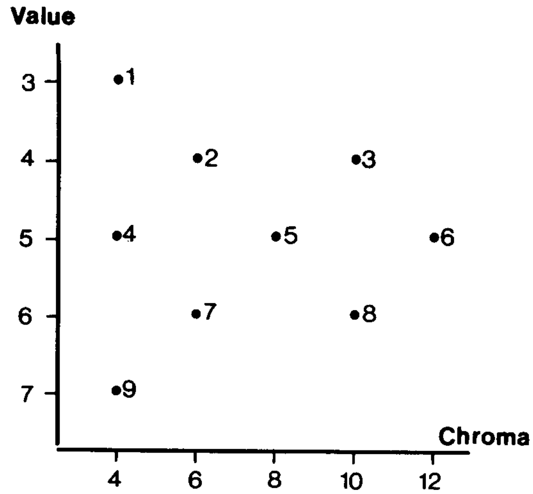

Figure 1. Design configuration for the nine Munsell colors.

suggesting that the use of fixed exponents in the analysis may not reveal the "best" fit of the data. In the present study, the analysis thus served to determine the "closest" fit of the data either by the city-block or the Euclidean model. A second factor of importance in the evaluation of similarity structure concerns the particular set of stimuli used in the experiment. The distance between two stimuli is not only a function of their similarity, but it also depends on the other stimuli in the set, indicating that the structure of the similarity space varies with the particular stimuli that compose the set (e.g., Crist, 1981; Monahan & Lockhead, 1977). Only one set of each stimulus category was used in the present study, which would not allow a separation of the effects of the stimulus class from those of the stimulus set. However, the purpose of this investigation was to compare similarity structures *within* each stimulus category, using stimulus sets that have already been extensively investigated; this approach offers some guarantee that differences in similarity structure as a function of experimental conditions would not reflect stimulus-set effects.

General Method

All the experiments were conducted using two-choice (*same-different*) reaction time tasks, with the two stimuli of a pair being presented simultaneously, one on each side of the center of the visual field.

Stimuli and Design

Four sets of stimuli were employed in these experiments. They were constructed according to the same criteria as those used in previous MDS studies.

Colors. The color stimuli were patches of nine Munsell 5R colors (red) similar to those used by Torgerson (1958). These patches were obtained from the Munsell Color Company, had a matte finish, and varied along two dimensions: value (brightness) and chroma (saturation). The design configuration of the color stimulus set upon its component dimensions is presented in Figure 1.

The color patches were mounted by pair, one to the left and one to the right of the center of gray (Munsell N5) 5 × 7-in. (12.7 × 17.8-cm) cards. When presented, each patch subtended a visual angle of 2.15° in height and 3° in width, with its center appearing 2° from the center of the visual field.

Circles. The second set of stimuli consisted of nine circles with a radial spoke in the right superior quadrant, similar in design to those used by Shepard (1964). The two dimensions along which the nine stimuli varied were the size of the radius (1.2, 1.6, and 2.0 cm) and the inclination of the radius (20°, 45°, and 70° from the vertical). They were drawn in black ink on a white background, and they appeared in pairs, each stimulus centered 2° from fixation, one in each visual field. Two design configurations of the same circle stimulus set are presented: in terms of circle size and radius inclination in Figure 2, and of length and angle of the spoke in Figure 3.

Parallelograms. Seven parallelograms, varying along the dimensions of size and tilt, were constructed so as to be exactly identical in shape and size to those used by Attneave (1950). The shorter sides of each parallelogram were oriented horizontally and were always half the length of the longer sides. They were drawn in black ink on a white background, and each stimulus of a pair appeared centered 2° from fixation, one stimulus in each visual field.

The design configuration of the parallelogram stimulus set upon its component dimensions (length of the longer side in centimeters and angle of tilt of the longer side with the horizontal in degrees) is presented in Figure 4.

Rectangles. The rectangle stimuli were 9 completely black rectangles on a white background, drawn from a larger set of 17 designed by Krantz and Tversky (1975). The 9 rectangles selected in this study comprised those with intermediary values in the two dimensions of length and width (i.e., Stimuli 9 to 17 of Krantz & Tversky, 1975). As a reference, Stimulus 9 was 1° high and 3.7° wide, and the inner edge of each stimulus of a pair was located 0.5° from fixation. Two design configurations of these stimuli are presented: In Figure 5, the rectangles are positioned in a two-dimensional space with real length and width as coordinates. As can be seen in this figure, distances separating each stimulus are not equal as the particular values of height and width were chosen by Krantz and Tversky so as to be equally spaced on a logarithmic scale. Figure 6 shows the design configuration of the rectangle stimuli as a function of log height (H) and log width (W), allowing equal interval

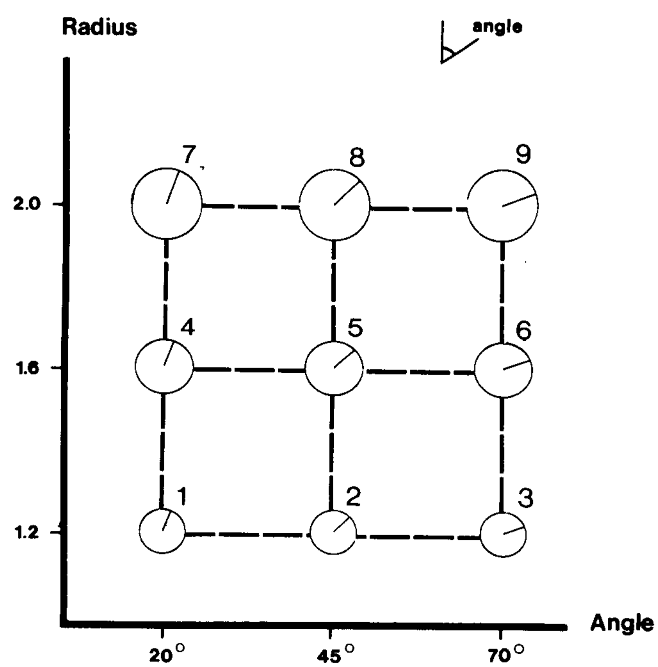

Figure 2. Design configuration for the nine circles in terms of radius and angle of the spoke.

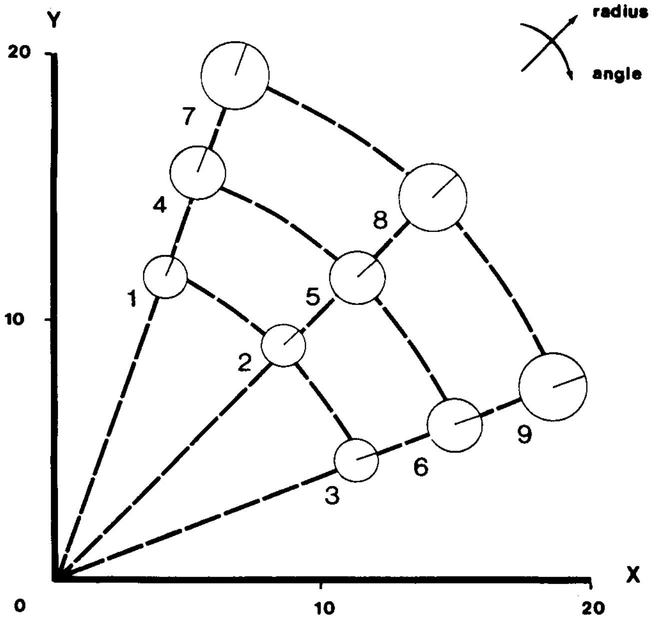

Figure 3. Design configuration for the nine circles in terms of (x,y) coordinates of the tip of the spoke.

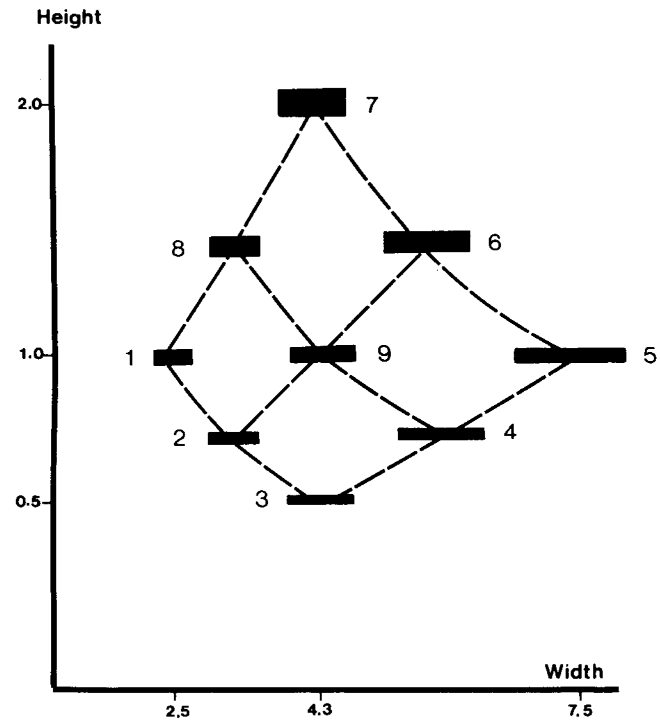

Figure 5. Design configuration for nine rectangles in terms of height and width.

between adjacent stimuli. Expressing variations along the two dimensions in a logarithmic scale allows the simultaneous representation of the set of rectangles along log area (W × H) and log shape (W/H) dimensions shown by the broken lines at 45° to the height and width axes.

Procedure

For each stimulus set, all possible *different* pairs were presented four times in an experimental session, along with an equal number of *same* pairs. There was no left–right counterbalancing of stimulus locations for *different* pairs. The task of the subject was to press one key if the two stimuli of a pair were the same and to press another key if they were different. The keys were placed one above the other along the midline axis in front of the subject who responded with the index and middle fingers of his right hand. Both speed and accuracy in responding were stressed in the instructions.

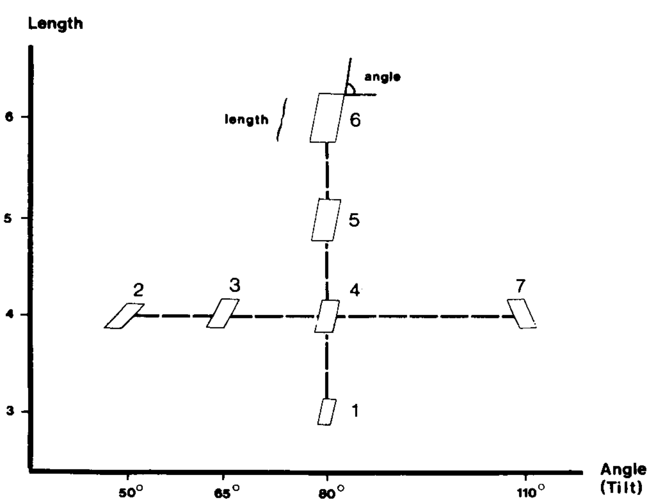

Figure 4. Design configuration for seven parallelograms in terms of side length and angle (tilt).

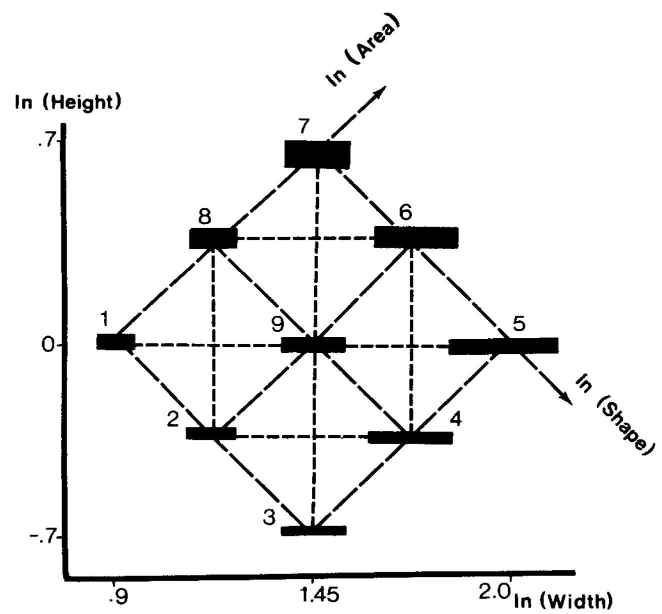

Figure 6. Theoretical configuration representing nine rectangles equally spaced in terms of logarithm (ln) height and width (in centimeters).

The color stimuli were presented through a Gerbrands T3-B-1 tachistoscope, for 50 ms at a luminance of 10 mL (34.26 cd/m^2) when measured for stimulus 5 (chroma 8 and value 5). The pre- and postexposure field was dark with a white dot in its center, which disappeared upon the presentation of the stimuli. Subjects were warned to fixate the central dot 1 s before stimulus presentation. Five sessions of 288 trials were run on consecutive days, the first session being used as practice. Order of presentation was randomly determined, except that the same color patch could not appear more than three times in succession and that a *same* or *different* type of response was not allowed to occur on more than 5 consecutive trials.

The circle, parallelogram, and rectangle stimuli were presented with different equipment while keeping the basic procedures unchanged. Stimulus presentation was made through a Kodak random-access projector on a translucent screen behind which the subject was seated about 80 cm away in a dark room. Stimulus presentation, exposure duration, subject's response accuracy and latency, and intertrial interval of 3 s were controlled by a PDP 11/20 computer. All stimuli appeared in black on a 10-mL (34.26 cd/m^2) white background. The pre- and postexposure field was dimly lit at 1 mL (3.43 cd/m^2). A trial started with a 500-ms tone to warn the subject to fixate the central dot. A pair of stimuli appeared 1 s after the tone onset, and the subject responded by pressing one key or the other according to the instructions.

Circles and rectangles were tested separately in one practice and four experimental sessions of 288 trials each, with an equal number of *same* and *different* trials randomly presented with the same restrictions as those prevailing with the color stimuli and with the first session used as practice. Sessions were run on consecutive days, at exposure duration of 50 ms for circles and 30 ms for rectangles. The seven parallelogram stimuli were tested in one practice session and two experimental sessions of 168 trials each, at an exposure duration of 50 ms.

Subjects

Two male subjects participated in the experiments. They were right-handed, with normal acuity and normal color vision as tested with the Ishihara color-blind test.

Analysis

All the computations were performed by MAXRT, a Fortran program designed to carry out the analyses described earlier. Both the city-block and the Euclidean metric were fitted in dimensionalities of two and three. The data of each subject and for each experiment were analyzed separately, using the reaction time of each trial of all the experimental sessions as input for each analysis.

One of the advantages of the maximum likelihood estimation employed in MAXRT is that it allows a computation of the Akaike Information Criterion (*AIC*; Akaike, 1974) for goodness-of-fit comparisons of the various models. This statistic is defined by

$$AIC; = -2 \ln L^* + 2n(p),$$

where L^* is the maximum likelihood of a fitted model and $n(p)$ the effective number of model parameters. The rationale is that the maximum log likelihood, which can be generally made larger by increasing parameters in the model, should be penalized by the number of parameters in the model in order to obtain a predictive measure of goodness of fit. Because $\ln L^*$ is multiplied by -2, a smaller value of the *AIC* indicates a better fitting model. This statistic may be used to compare any models and/or any number of models simultaneously. This is a definite advantage of the *AIC* statistic over the more conventional asymptotic chi-square goodness-of-fit test derived from the likelihood ratio principle, which is restricted to a comparison of two nested models at a time. Nonetheless, in case where both the *AIC* and the asymptotic chi-square test are applicable, they tend to yield identical results. In this study, the *AIC* statistic will consistently be used, because some of the comparisons to be made involve nonhierarchical models (e.g., comparisons between different metrics). The *AIC* statistic was developed in such a way that the model with the smaller *AIC* value, however small the difference between the values of two solutions, can reliably be considered the better fitting model. The multiplication of $\ln L^*$ by -2 and the addition of twice the number of model parameters are by no means arbitrary and follow the entropy maximization principle. A detailed discussion and several examples of the effective use of this statistic in psychometrics are presented in Akaike (1977), Takane (1981), and Takane and Sergent (1983).

Table 1
Summary Statistics for Color Data

Dimensionality & metric	Subject 1 Uncon	Subject 1 Con	Subject 2 Uncon	Subject 2 Con
Two-dimensional				
Euclidean				
AIC	28.6	44.2	67.0	72.6
$n(p)$	15	8	15	8
City block				
AIC	28.2	46.6	62.8	63.8
$n(p)$	16	8	16	8
Three-dimensional				
Euclidean				
AIC	11.8[a]		20.4[a]	
$n(p)$	21		21	
City block				
AIC	25.2		57.0	
$n(p)$	24		24	

Note. AIC = Akaike Information Criterion. Uncon = unconstrained; Con = constrained. $n(p)$ = number of parameters.
[a] The minimum *AIC* solution.

Results

In order to determine the most appropriate models, each subject's data from all the experimental sessions for each set of stimuli were analyzed seperately under constrained and unconstrained conditions and fitted by the city-block and Euclidean distance models. In each case, the *AIC* statistic was used to identify the best fitting model, that is, the solution providing the minimum *AIC* value. These statistics will be presented for each stimulus set, along with the effective number of parameters in the model. The derived configuration of the best solutions will also be presented and described.

Color Stimuli

The summary statistics of the color data are presented in Table 1. The "constrained" solution aims at testing the hypothesis that the psychological distances separating each stimulus of the color set essentially correspond to the intervals in value and chroma between each member of the set as defined in the Munsell color systems (see Figure 1). Thus, the constrained MDS solution provides a representation that is congruent with the distances specified by the Munsell company. A comparison of the *AIC* value of this solution with that of the "unconstrained" solution (that is, without the restrictions imposed by

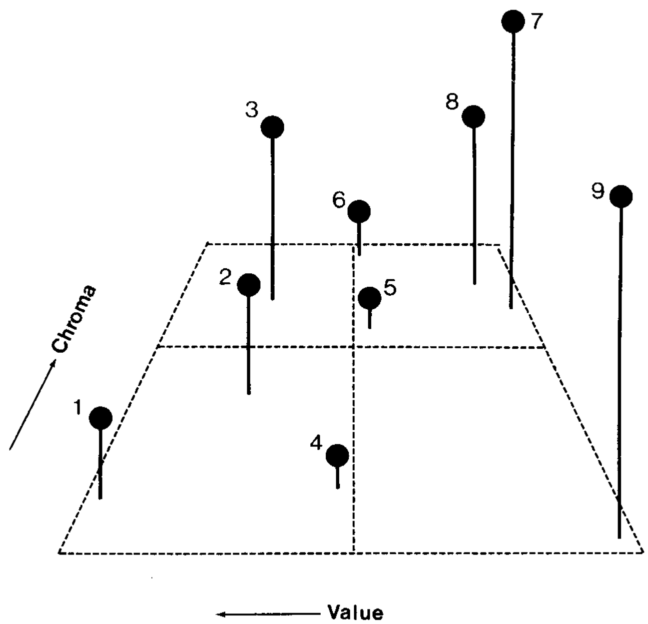

Figure 7. Derived three-dimensional configuration for colors for Subject 1.

the Munsell system) may then determine whether or not the psychological distances between the nine color stimuli are best described by the intervals of the Munsell system.

The results shown in Table 1 indicate that for both subjects the three-dimensional unconstrained Euclidean solution provides the best fit of the data. This indicates that the dimensionality of the representation space is at least three. The *AIC* values are 11.8 and 20.4 for Subjects 1 and 2, respectively, and considerably lower than any of the two-dimensional solutions. The 2 subjects did not perceive the dissimilarities between the color stimuli presented here in the manner specified in the Munsell system. The fact that the Euclidean metric best represents the psychological distances between the color stimuli is consistent with previous research (e.g., Hyman & Well, 1968; Takane, 1981; Torgenson, 1958) based on different data-collection procedures. However, finding that the dimensionality is at least three in the particular procedure employed in this experiment indicates that the dimensions may interact in ways different from those used under unlimited viewing conditions in which a two-dimensional solution has been found.

The derived configurations of the three-dimensional solutions for the 2 subjects are presented in Figures 7 and 8. The stimuli are identified by numbers as in Figure 1. The horizontal direction roughly corresponds with the value dimension, and the depth direction with the chroma dimension. The third dimension emerging in these solutions is described in the vertical direction and has the effect of bending downward the horizontal direction. The value dimension is thus not uniform and takes a concave shape so that the extreme values are closer to one another than would be predicted from the design configuration. No such distortion was obtained for the chroma dimension, as shown by Stimuli 4, 5, and 6, which lie essentially on a flat plane. It thus appears that the bending of the value dimension prevented the constrained solution from providing the best fit of the data, and this bending was sufficiently pronounced to yield a better approximation by a three-dimensional solution despite the increased number of parameters required. Although the tachistoscopic mode of presentation of the color stimuli may account for the distortion of the brightness dimension, the exact nature of this phenomenon remains unspecified. Takane (1982) found a similar bending of the color configuration by using the method of triadic combinations for dissimilarity judgments. One possibility (suggested by a reviewer) is that the third dimension (a bending of the value dimension) resulted from the particular value of the background. This background was a Munsell gray with a value of 5, and it is noteworthy that Stimuli 4, 5, and 6 were also a Munsell value of 5 (see design configuration, Figure 1). This particular background was chosen following current practice in research on color perception (e.g., Chang & Caroll, 1980), but it may have interfered with the evaluation of similarity structure of the color space.

Aside from the bending of the configuration, however, the general layout of the configuration is not radically different from the Munsell configuration. The only exception is that Stimulus 7 is positioned at a somewhat odd place (i.e., too high in chroma dimension), and this finding is consistent for both subjects.

Circle Stimuli

Table 2 presents the summary statistics for the analysis of reaction times to compare *different* circle stimuli depicted in Figure 2. The data were analyzed following the same procedure as for the color data, and both constrained and unconstrained

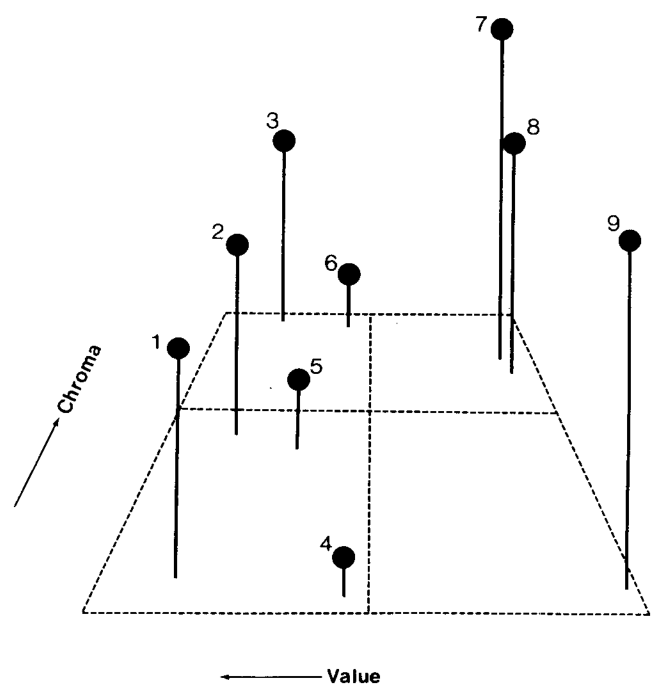

Figure 8. Derived three-dimensional configuration for colors for Subject 2.

solutions were obtained for two- and three-dimensional city-block and Euclidean models. The sets of restrictions imposed on the two-dimensional solution were made in terms of radius length and radius inclination as described in Figure 3.

The results displayed in Table 2 indicate individual differences between the subjects, both in terms of best dimensionality and of restrictions on the distance model. Data of Subject 1 were best approximated by the constrained three-dimensional Euclidean solution as shown by the *AIC* value, which was the lowest of all solutions. The derived configuration of this solution is presented in Figure 9, with the radius as one dimension, and the angle of the spoke emerging as two dimensions resulting from distortion of its original plane. Although the distances between circles increase as a function of an increase in size, distances as a function of angle of the radius do not parallel the intervals in terms of degree of inclination. The perceived distance between inclination of 20° and inclination of 70° is shorter than would be predicted from the design configuration.[1]

On the other hand, the best fit of data for Subject 2 was obtained in the unconstrained two-dimensional Euclidean solution, the derived configuration of which is shown in Figure 10. The two dimensions are identified as size of the radius and angle of inclination, and they combine in ways different from that depicted in the design configuration. For one thing, the distance separating circles varying in radius inclination is not the same for equal difference in inclination. For example, a difference of 25° is perceived as smaller between 20° and 45° than between 45° and 70°.

For both subjects, it is fairly obvious that the best fitting configurations do not conform to the design configuration depicted in Figure 3, although because of some technical difficulties this is not formally tested. It should also be emphasized that the differences between the 2 subjects' configurations are not so great as they may look. They are topologically quite similar, and the dimensionality difference is created by bending of the two-dimensional configuration in Subject 1.

Table 2
Summary Statistics for Circle Data

Dimensionality & metric	Subject 1		Subject 2	
	Uncon	Con	Uncon	Con
Two-dimensional				
Euclidean				
AIC	50.4	53.2	44.4[a]	51.2
n(p)	15	4	15	4
City block				
AIC	44.8	48.8	59.0	66.2
n(p)	16	4	16	4
Three-dimensional				
Euclidean				
AIC	39.6	38.0[a]	45.2	
n(p)	21	5	21	
City block				
AIC	45.2	45.0	53.0	
n(p)	24	5	25	

Note. AIC = Akaike Information Criterion. Uncon = unconstrained; Con = constrained. *n(p)* = number of parameters.
[a] The minimum *AIC* solution.

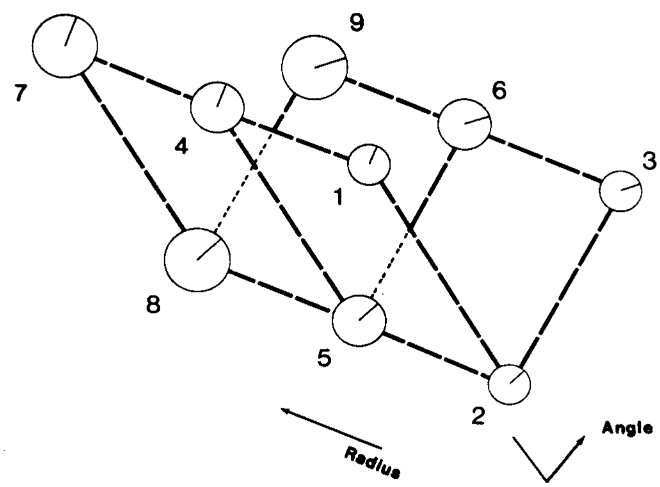

Figure 9. Derived three-dimensional configuration for circles for Subject 1 (constrained solution).

The present findings diverge from those of previous research, which had shown that the perceived distances between these geometric stimuli were best represented by a city-block metric model (Hyman & Well, 1968; Shepard, 1964). The results of the 2 subjects illustrate that even if the two dimensions are separable, they do not combine by simple summation of the differences along each dimension. Deviations from the city-block metric in the solutions of both subjects suggest that in the present experimental conditions the two dimensions are not perceived as purely separable.

Parallelogram Stimuli

Summary statistics of the analysis of reaction times to compare the *different* parallelogram stimuli depicted in Figure 4 are shown in Table 3. The constrained solution was restricted in terms of length and tilt of the longer sides, as shown in the design configuration presented in Figure 4. For both subjects, the two-dimensional city-block metric solution provided the best representation of the dissimilarity data. However, the 2 subjects diverged in the particular dimensions on which they based their judgments. Although the unconstrained solution provides the best fit for Subject 1, the psychological distances of Subject 2 were congruent with the particular intervals in length and tilt between each member of the parallelogram stimulus set.

The derived configuration of Subject 1's two-dimensional so-

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[1] A constrained three-dimensional solution provided the best fit of the data in spite of only two physical dimensions being put as constraints. The third dimension resulted from a bending of the inclination dimension, and the departure from the constraints was not so pronounced as to prevent the program from reaching a three-dimensional solution. Because a constrained solution requires fewer parameters to be estimated, the *AIC* value was the smallest for this solution even though the maximum likelihood (which is multiplied by −2) was not as high as that of the other solutions. In no other set of stimuli, for any subject, was the program capable of reaching a three-dimensional constrained solution.

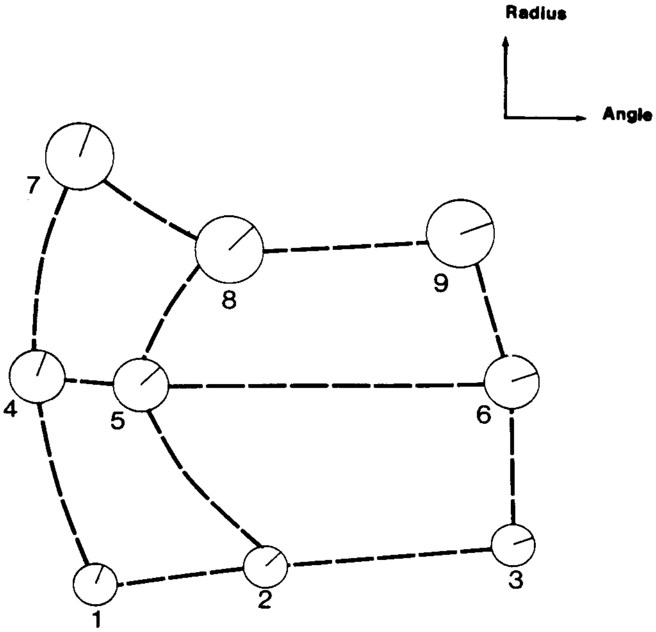

Figure 10. Derived two-dimensional configuration for circles for Subject 2.

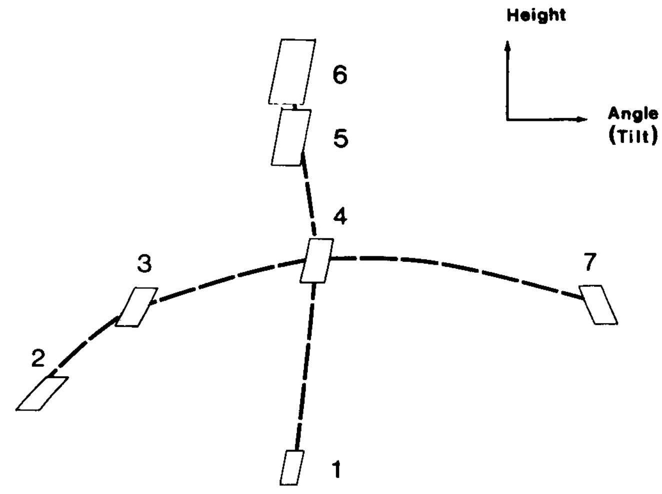

Figure 11. Derived two-dimensional configuration for parallelograms for Subject 1.

lution is presented in Figure 11. The departure from predictions based on length and tilt is evidenced by the curved layout of the Parallelograms 2, 3, 4, and 7, which all have the same side length. In this configuration, these parallelograms are located in space as a function of their height instead of the length of the longer sides. In comparison, Subject 2's derived configuration shown in Figure 12 represents the two-dimensional solution constrained in terms of length and tilt, which yielded the best approximation of the data.

For both subjects, the city-block metric proved the appropriate spatial representation of these geometric stimuli, which is consistent with previous findings reported by Attneave (1950), Hyman and Well (1968), and Dunn (1983). This indicates that even in the present experimental conditions the separability of the two dimensions inherent in this stimulus set resulted in the perceived differences among the stimuli being the summated difference along each dimension.

Rectangle Stimuli

Table 4 displays the summary statistics of the analyses of the reaction times to compare the *different* rectangle stimuli. Two types of restrictions could be imposed on the models, according to two different combinations of stimulus dimensions: width and height (W, H), area (A = W × H) and shape (S = W/H). These two types of constraints correspond to the two design configurations depicted in Figures 5 and 6. Neither subject perceived the differences between the rectangles along the restrictions imposed on the constrained solutions, however, and indi-

Table 3
Summary Statistics for Parallelogram Data

Dimensionality & metric	Subject 1		Subject 2	
	Uncon	Con	Uncon	Con
Two-dimensional				
Euclidean				
AIC	62.2	71.0	84.0	78.2
n(*p*)	11	6	11	6
City block				
AIC	60.8[a]	64.6	78.4	76.8[a]
n(*p*)	12	6	12	6
Three-dimensional				
Euclidean				
AIC	65.0		78.0	
n(*p*)	15		15	

Note. AIC = Akaike Information Criterion. Uncon = unconstrained; Con = constrained. *n*(*p*) = number of parameters.
[a] The minimum *AIC* solution.

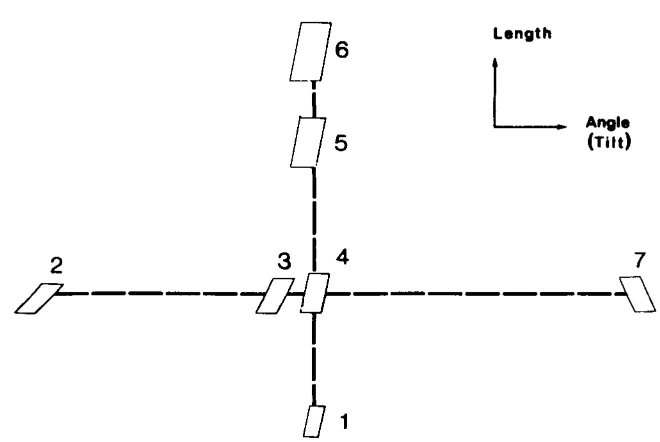

Figure 12. Derived two-dimensional configuration for parallelograms for Subject 2 (constrained solution).

Table 4
Summary Statistics for Rectangle Data

Dimensionality & metric	Subject 1 Uncon	Subject 1 Con	Subject 2 Uncon	Subject 2 Con
Two-dimensional				
Euclidean				
AIC	31.6		42.0	
A × S		92.0		66.2
n(*p*)	15	4	15	4
W × H		66.0		91.4
n(*p*)		8		8
City block				
AIC	26.0[a]		40.8	
A × S		89.9		70.2
n(*p*)	16	4	16	4
W × H		100.4		111.4
n(*p*)		4		8
Three-dimensional				
Euclidean				
AIC	29.2		31.8	
n(*p*)	21		21	
City block				
AIC	31.4		22.6[a]	
n(*p*)	24		24	

Note. AIC = Akaike Information Criterion; *n*(*p*) = number of parameters; A × S = Area × Shape; W × H = Width × Height; Uncon = unconstrained; Con = constrained.
[a] The minimum *AIC* solution.

vidual differences emerged again. The data of Subject 1 were best represented in the form of a two-dimensional city-block model, whereas a three-dimensional city-block model provided the best fit for the data of Subject 2.

The derived configuration of Subject 1's solution is shown in Figure 13. The rectangles are located in this space roughly along the dimensions of area and shape, and the perceived distances depart from the physically equal distances separating the stimuli when represented in terms of area and shape (see Figure 6). For one thing, the interval between rectangles tends to become larger as the area increases. For another, the shape dimension differentially affects the contribution of the area dimension to perceived dissimilarities. This interaction is clearly illustrated in the bottom left part of Figure 13 (e.g., *d*[1, 8] compared with *d*[2, 9]). A similar pattern is present in the derived configuration of Subject 2, shown in Figure 14. However, as indicated by the *AIC* for this subject in Table 4, the three-dimensional city-block solution provided the best fit of the data, the third dimension resulting from a twisting and bending of the plane across the area and shape dimensions. Thus, whereas "taller" rectangles are sticking out of the area-and-shape plane for rectangles with large area (5, 6 and 7), the opposite seems to be true of small rectangles (all other rectangles). However, the significance of this depth dimension is not immediately clear.

Additional Analyses

The preceding analyses of reaction-time data have revealed discrepancies with previous findings obtained on the same stimulus sets under different experimental conditions, as well as individual differences. In order to examine whether the departure from typical findings was due to idiosyncrasies of the subjects, these 2 subjects were further tested on the same stimulus sets, using dissimilarity judgments as dependent measure. This served the double purpose (a) of establishing which representational space would best describe the similarity structure derived from the dissimilarity judgments in conditions more like those prevailing in previous studies and (b) of constraining the analyses of reaction-time data with the configurations obtained from the dissimilarity judgments. A detailed description of this additional series of experiments is beyond the scope of this study

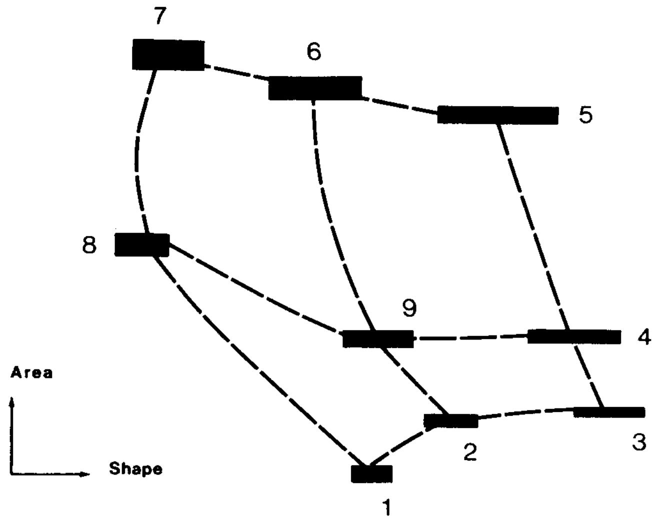

Figure 13. Derived two-dimensional configuration for rectangles for Subject 1.

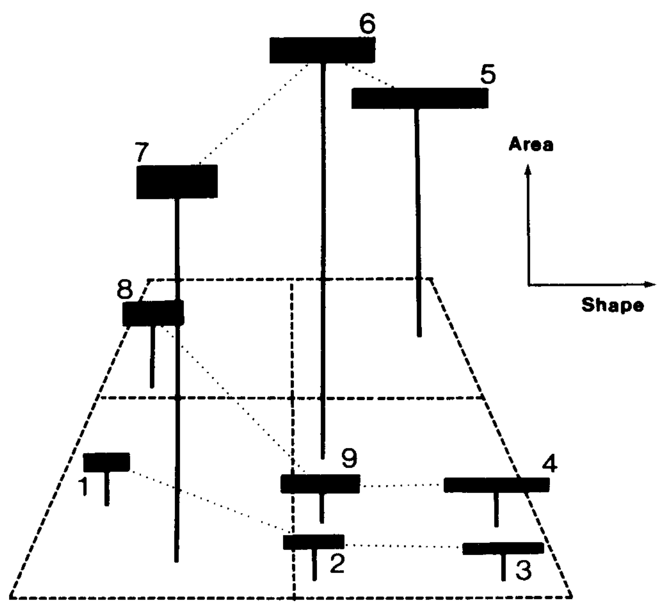

Figure 14. Derived three-dimensional configuration for rectangles for Subject 2.

and will be presented in a forthcoming article (Sergent & Takane, 1986).

The main methodological difference between this experiment and the previous series of experiments involved the mode and nature of the response while the subjects, equipment, and stimuli were exactly the same in all respects. The following changes were introduced: Only *different* pairs were presented, and the subject responded by rating the degree of dissimilarity between the two stimuli of each possible *different* pair, on a scale of 1 to 25 (1 for *highly similar* and 25 for *highly dissimilar*). The stimuli were presented at the same exposure duration as in the main experiments in one condition, and, in the other condition, remained on the screen until the subject responded. There was no time limit for the production of a response in either condition. There were four sessions for each of the four stimulus sets at each exposure duration, resulting in eight experimental conditions (of four sessions each) conducted on consecutive days.

The dissimilarity judgments from each subject, for each stimulus set and at each exposure duration, were separately scaled by the program KYST-2A (Kruskal, Young, & Seery, 1978), using the city-block distance and the Euclidean distance formulae for two- and three-dimensional solutions. For the purpose of this study, only the goodness of fit of the dissimilarity data for each solution was of interest. This was estimated by the "stress" of the solution, which is a measure of how poorly the regression function relating similarity to distance fits the dissimilarity judgments: Thus, the lower the stress, the better the fit. The stress value of each solution is shown in Table 5, where the lowest stress value for each experimental condition and for each subject is underlined. These results essentially conform to earlier findings that a city-block distance model better describes the similarity structure of geometrical shapes, whereas Euclidean space better represents color stimuli. There were two exceptions to this general trend, however: The short exposure for rectangles for Subject 1 and the long exposure for circles for Subject 2 yielded a better fit of the data by the Euclidean distance model. Overall, the results from the 2 subjects are qualitatively similar to those typically obtained in previous studies based on dissimilarity judgments, and they suggest that the discrepancies noted in the analysis of reaction-time data are probably not due to the subjects per se, but more likely to differences in mode of responding.

To further examine this suggestion, the reaction-time data were analyzed again, using as constraints the final configurations of the MDS analyses of the dissimilarity judgments. If the particular experimental conditions have no influence on the perceived similarity structure of a stimulus set, constraining the analysis of each subject's reaction times by the configuration of his best solutions in the dissimilarity judgments should result in a very close fit as indicated by small *AIC* values. By contrast, if the experimental conditions play a critical role in determining the perceived similarity between stimuli, the constraints imposed on the analysis of the reaction-time data should produce a poor fit of the data, as indicated by relatively high *AIC* values.

Table 6 presents the *AIC* values of the MDS solutions for reaction-time data, constrained by the final configuration derived from the analysis of the dissimilarity judgments of each subject in each experimental condition. The analysis of the reaction times was conducted only for the dimensionality that yielded the best solution in the earlier series of analyses. As can be seen in this table, the *AIC* values were much higher than those shown in Tables 1–4, in all conditions, suggesting a poor fit between the similarity spaces derived from reaction times and dissimilarity judgments, even when the same mode of stimulus presentation prevailed in the two experimental conditions. Only when constrained by the configuration derived from the short exposure of circles for Subject 1 did the goodness of fit approach that obtained in the previous analyses. This poor correspondence between the configurations derived from the same stimulus sets and the same subjects seems to indicate that the similarity structure of a stimulus set is better predicted by knowledge of the objectively manipulated dimensions than by knowledge of the similarity structure of this set derived from performance in different experimental conditions. This suggestion must be qualified, however, and may apply to relatively unfamiliar stimuli for which subjects may not have developed a well established strategy for processing them. In fact, Podgorny and Garner (1979) reported a close spatial correspondence between similarity structures of letters of the alphabet derived from reaction times and dissimilarity ratings. The use of highly familiar stimuli may partly account for this finding. More important, however, Podgorny and Garner (1979) opted for a common two-dimensional Euclidean space to represent the two sets of data, and it was not the aim of their study to determine whether this particular space provided the best fit of the data. It is therefore possible that a different space in a different dimensionality would have emerged for each set of data had a best-fitting procedure been used.

The main purpose of this additional study was to establish that discrepancies noted in the main analyses were not due to some particularities of the subjects, and this was actually confirmed. On the other hand, there was no indication that a change in exposure duration consistently resulted in a change in the goodness of fit of the dissimilarity judgments (see Table 5). Moreover, the additional analyses of reaction times did not yield a better fit when constrained by configurations derived from short exposure presentations (as in the main experiments) than when constrained by configurations obtained under long exposure presentations (see Table 6). The fact that both exposure conditions resulted in poor correspondence between similarity structures derived from reaction times and dissimilarity judgments precludes any definitive suggestions regarding the effect of state limitations on perceived similarity. It may, then, be the mode (speeded vs. untimed) and the nature (two-choice *same–different* decision vs. dissimilarity rating) of the response that determines much of the difference between the resulting similarity structures in the two experimental conditions. The data of these additional experiments will not be further discussed here, and the following discussion will proceed on the basis of the main analyses of reaction-time data.

Discussion

The present experiments were designed in an attempt to carry out MDS analyses of reaction-time data in the context of *same–different* judgments in order to uncover the relevant dimensions on which comparisons are made, to examine the rules governing the combination of these dimensions, and to

Table 5
"Stress" Value of the Two- and Three-Dimensional Solutions for City-Block and Euclidean Metric, Derived From Dissimilarity Judgments

Stimulus set, exposure, & dimensionality	Subject 1		Subject 2	
	City block	Euclidean	City block	Euclidean
Rectangles				
Short exp.				
2-D	.181	.159	.243	.187
3-D	.083	.074	.098	.120
Long exp.				
2-D	.172	.170	.235	.230
3-D	.077	.080	.100	.129
Circles				
Short exp.				
2-D	.211	.204	.63	.168
3-D	.112	.129	.107	.120
Long exp.				
2-D	.136	.157	.175	.151
3-D	.089	.102	.094	.085
Parallelograms				
Short exp.				
2-D	.084	.122	.159	.197
3-D	.055	.100	.065	.128
Long exp.				
2-D	.083	.119	.106	.121
3-D	.062	.074	.021	.079
Colors				
Short exp.				
2-D	.133	.102	.226	.170
3-D	.093	.075	.130	.113
Long exp.				
2-D	.116	.099	.147	.137
3-D	.095	.081	.110	.106

Note. Lowest "stress" value within exposure condition for each subject is underlined. Exp. = exposure. 2-D = two-dimensional; 3-D = three-dimensional.

determine the representational space for each stimulus set as it can be derived from performance under state- and resource-limitations. Several aspects of the results will be discussed in turn, first comparing the present solutions with those obtained in unlimited response conditions and then examining the issue of integrality and separability of dimensions and the problem of the mapping of physical onto psychological dimensions.

Comparison With Previous MDS Studies

One of the motivations that led to the development of the present MDS procedure was to close the gap between studies of similarity relations among visual patterns and chronometric studies of information processing (Takane & Sergent, 1983). Most experiments that have examined the similarity structures of various stimulus sets were carried out under conditions different from those prevailing in usual RT experiments, yet findings from the former experiments are often used to guide interpretations of results from the latter (e.g., Cheng & Pachella, 1984; Sergent, 1984), even though the two types of study may result in different perceived similarity structures. The present approach offers the possibility of comparing the particular similarity structures of several stimulus sets derived from dissimilarity judgments performed under these different experimental conditions.

The most direct comparison is provided by the goodness of fit of each solution of each subject in each of the stimulus sets for the constrained and the unconstrained analyses. First, the *AIC* provides an index of goodness of fit that takes into account the number of parameters required to achieve the best approximation of the data and thus offers a guarantee of parsimony and reliability. In addition, the constraints imposed on the representational model were based on objective variations among the stimuli of a set or on the configuration derived from MDS analyses of dissimilarity judgments. Thus, a better fit for the constrained solution would suggest similar representational space for a given set across different experimental conditions, that is, independent of the way the data were collected, and would then indicate a stable and invariant similarity structure for this particular set. On the other hand, a better fit for the unconstrained solution would suggest that the combination rules and/or the relevant dimensions are not invariant across experimental conditions.

The present results indicate that the representational space of the four stimulus sets differed to some extent from the solution arrived at in previous studies. In only two out of eight cases did the objectively constrained solution provide the best fit of the data; in one of the two the solution departed from prescribed dimensionality. These divergences from previous findings were

Table 6
AIC Values of Multidimensional Scaling Solutions in Additional Analyses

Stimulus set, exposure, & dimensionality	Subject 1 City block	Subject 1 Euclidean	Subject 2 City block	Subject 2 Euclidean
Rectangles				
Short exp.				
2-D	285.6	234.8		
3-D			163.9	199.1
Long exp.				
2-D	297.5	264.7		
3-D			235.9	185.6
Circles				
Short exp.				
2-D			174.5	144.6
3-D	63.5	55.7		
Long exp.				
2-D			119.4	103.7
3-D	89.3	71.2		
Parallelograms				
Short exp.				
2-D	132.1	141.2	107.3	105.8
Long exp.				
2-D	145.9	124.8	112.2	108.5
Colors				
Short exp.				
3-D	286.4	203.7	391.5	286.4
Long exp.				
3-D	317.5	260.9	341.4	259.1

Note. AIC = Akaike Information Criterion. Exp. = exposure. 2-D = two-dimensional; 3-D = three-dimensional.

not uniform across stimulus sets and across subjects, and they occurred at the level of the dimensionality of the representational space, the combination rules, or the relevant dimensions. However, the combination rules (the best metrics) were the same across the 2 subjects for each specific stimulus set. As expected, the color stimuli were best represented in a Euclidean space, but in a dimensionality of three instead of two. The addition of a third dimension did not arise from a large departure from the usual two-dimensional representation and reflected only a slight bending of the value direction, possibly due to the value of the background, a finding qualitatively similar to that obtained by Takane (1982) in an experiment based on the method of triadic combinations for dissimilarity judgments. These results, therefore, are in general agreement with the suggestion that a Euclidean space best describes the similarity relations of color stimuli (Hyman & Well, 1968; Shepard, 1964) and that interactions between component dimensions may occur under some circumstances within this representational space.

By contrast, the results for the circle stimuli showed departure from the usual combinatorial rule typical of geometric forms. For both subjects, the Euclidean solution provided the best approximation of the data, and the radius-inclination dimension was not perceived as would be expected of an attribute that is perceptually distinct of the other dimension. This finding is not consistent with the view that geometric forms have separable dimensions, and it suggests that the particular rules governing the dimensional decomposition of these forms may vary depending on the viewing and response conditions under which data are collected. This suggestion concurs with Smith and Baron's (1981) finding of different and uncorrelated interference of an irrelevant dimension on the classification of integral stimuli in timed and untimed tasks. In this sense, the development of an MDS procedure for reaction-time data may prove useful in providing information about similarity structures of stimulus sets as they actually emerge under conditions of speeded judgment process. In terms of space dimensionality, this issue is further complicated, however, by the results on the rectangle stimuli that yielded different best solutions for the 2 subjects. In addition, the solutions did not strictly conform to the prescribed constraints of shape and area, and they revealed interactions between these two dimensions, as already observed by Krantz and Tversky (1975). Still another pattern of results was provided by the parallelogram data, showing that the rules by which components of some geometric forms are combined may be stable across experimental conditions and task requirements (see Ward, Foley, & Cole, 1986, for similar observations). The typical two-dimensional city-block space yielded the best description of the parallelogram stimuli in the present experiment, in agreement with previous studies (e.g., Attneave, 1950; Hyman & Well, 1968), even though the 2 subjects did not attend to the same set of dimensions in making their judgment.

These results do not allow any simple and direct conclusions to be drawn with respect to the effect of experimental conditions on the similarity structures of stimulus sets. All stimuli were not equally sensitive to procedural manipulations, and at the moment there does not seem to exist a rule that would permit predictions as to the properties of a stimulus set in different experimental conditions. This may then make it still more compelling to carry out MDS analyses of stimulus sets for which knowledge of the relevant component dimensions may be crucial in the design of an experiment. It also appears that subjects do not rely on the same attributes in comparing stimuli, which makes the evaluation of performance as a function of stimulus components somewhat unreliable if the relevant dimensions are simply assumed.[2] In addition, gathering some information concerning how the different components of multidimensional stimuli are combined in a particular situation may be useful because the present results indicate that integral and separable properties of some stimulus sets may not be stable across a variety of conditions.

Integrality and Separability of Stimulus Dimensions

The specification of the representational space best describing similarity relations among stimuli of a set has been used as part of a group of converging evidence toward characterizing the nature of the component dimensions of stimuli. Certain stimulus dimensions are phenomenologically distinct, whereas others are perceived as a unitary entity, and these properties have been conceived as referring to the separability and integrality of internal representations of visual objects. In addition to yielding different patterns of results in filtering, classification, and condensation tasks (see Garner, 1974), integral and separable stimuli are best embedded in spaces of different metrics. Thus, a city-block space better describes the representation of separable stimuli, whereas the Euclidean space provides the better representation of integral stimuli. The different distance metrics (i.e., the values of the Minkowski power) underlying the representational space of these two categories imply different rules governing the combination of their component dimensions, and these particular rules may refer to the way information is extracted from visual objects, depending on the dimensional nature of the stimuli. A Euclidean space implies that interpoint distance equals the square root of the sum of the squares of the differences on each dimension, which is unaffected by rotation of dimensional axes. By contrast, interpoint distance in a city-block space equals the sum of the absolute values of the differences on each dimension, and perceived dissimilarity is judged by determining the values of the stimuli on each of a fixed set of dimensions. Thus, dimensions do not contribute independently to judgments of overall dissimilarity in a Euclidean space, whereas the amount of information actually processed corresponds to the sum of the information contained in the component dimensions in a city-block space.

Although these views have gained support from experimenters using converging operations to specify the type of psychological structure of visual representations, there is also some difficulty in consistently accounting for all findings along these lines. Cheng and Pachella (1984) have recently disputed the validity of the concepts of integrality and separability, and they suggested that separable dimensions correspond to psychological attributes, that is, those dimensions that can be selectively attended to, whereas dimensions that do not correspond to psychological attributes are "inseparable." They supported this suggestion of dimensional separability in three experiments with geometric forms that examined the pattern of interference between dimensions that were either "psychological" or "nonpsychological." The present results bear on this suggestion, which makes specific predictions with respect to the representational space of stimuli depending on whether or not the relevant attributes are psychological.

One of the interesting features of MDS is its capacity to reveal the dimensions that are psychologically relevant in a stimulus set without any a priori commitment as to which dimensions are critical—in contrast to such tasks as classification and condensation used by Cheng and Pachella (1984) in which subjects are instructed to attend to a specific "relevant" dimension. The present findings offer several indications that this critique of the notions of integrality and separability may not be entirely well founded. For one thing, the suggestion that a "psychological" dimension is a dimension that can be selectively attended to implies that the best solution could never be the Euclidean solution, that is, a model providing the best description of integral or "inseparable" stimuli. The results presented earlier showed that this was not the case, not only for color stimuli but also for geometric forms such as Shepard's circles. "Psychological" dimensions need not be separable, and their relevance as a basis for processing does not prevent them from interacting with other dimensions. This is probably best illustrated by findings from research on face perception and recognition: Although the eyes are certainly a "psychological" dimension, their particular shape influences how other dimensions are perceived, and faces are best represented in a Euclidean space (Takane & Sergent, 1983). In addition, despite the psychological relevance of facial features, results from recognition studies reveal a "surprising inability of subjects to discriminate between individual features when they are embedded in the overall face" (Davies, Ellis, & Shepherd, 1977, p. 268). The problem, then, should be to specify when and why some psychological dimensions appear to be integral or separable and to consider the various experimental factors and individual idiosyncrasies that may determine such a variability.

Cheng and Pachella (1984) also took as an example of a "hypothetically nonpsychological dimension" the height of parallelograms that vary in tilt and length of the longer side (see Figure 3). They suggested, and later verified with the height of triangles, that this nonpsychological dimension was not perceived as separable and should therefore be considered as an "inseparable" dimension that cannot be attended to selectively. The results of Subject 1 in the present experiment (see Figure 11) stand in sharp contrast to this suggestion. Not only was the height of the parallelograms a psychologically relevant dimension, but a city-block representational space provided the best description of the similarity relations among parallelograms for this subject. Thus, in spite of Cheng and Pachella's (1984) re-

[2] Because only 2 subjects participated in this study and because their data were analyzed separately, individual differences were quite conspicuous. It must be noted, however, that the same combination rules underlay the data of both subjects in each stimulus set of all the reaction-time tasks and of all but two dissimilarity-judgment tasks. Studies in visual information processing usually consider group performance, and the separate analysis of each subject's data in the present study may have brought to the fore individual differences in perception that are inherently present but not investigated in any visual experiment.

sults in support of their own suggestion, the notion of psychological attribute as a basis for the separability of dimensions does not appear very compelling.

It is nonetheless the case, as these authors pointed out, that the concepts of integrality and separability remain often vague and are not consistently supported by empirical evidence; this situation has led to ad hoc refinements of their definition and application that reduce their explanatory power. Indeed, another attempt at circumscribing the characteristics of integral dimensions was made by Dunn (1983), who suggested that only homogeneous dimensions of multidimensional stimuli (that is, those dimensions measured in the same units within a stimulus, such as height and width of rectangles as opposed to size and angle of Shepard's circles) could be represented in a Euclidean space. The present results do not support this suggestion, and, despite their heterogeneity, the dimensions of size and angle of Shepard's circles were best embedded in a Euclidean space for both subjects (see Figures 9 and 10). This, however, should be seen as a healthy complication in understanding the properties of these concepts that have provided research on visual information processing with useful operational criteria for specifying stimulus structures and underlying operations.

Psychological Mapping of Physical Dimensions

Perceiving multidimensional stimuli necessarily involves mutual interplay between the stimulus and the observer, and, as noted in the introduction, the two sides of this interaction may be influenced by various factors that determine the representation of information and the type of operation that can be implemented. Among the variety of attributes that compose a stimulus, only a portion of them are psychologically relevant, and the present results suggest that this dimensional relevance is not invariant and may not be specified a priori. In addition, the concepts of integrality and separability, in reference to the representational space in which a stimulus set is best embedded, do not simply apply to the stimuli as such but also to the particular representation/operation generated by the observer as a function of the viewing and response conditions prevailing in the experimental situation. Little can be inferred from the present results in terms of principles that would predict the performance of a subject in specific conditions, but some trends may nonetheless be suggested.

Compared with results from previous studies, the present findings indicate a tendency for stimuli, the dimensions of which are usually perceived as separable, to be treated as integral with speeded judgment process. This was specifically the case of the circle stimuli for which distances between each instance of the set were best described by the Euclidean metric. This finding is consistent with Lockhead's (1972) suggestion that stimuli are initially perceived as integral "blobs" before the component dimensions become perceptually distinct. Thus, in conditions of time pressure to produce a response, the processing organism may operate on such an integral representation of information, provided such a representation is adequate for an efficient judgment. However appealing this explanation may be, it falls short of accounting for the finding that parallelograms, which were presented in similar experimental conditions, were best described in a city-block representational space. There is no obvious reason why parallelograms could not be compared on the basis of their overall similarity, and it would appear even more plausible for parallelograms than for circles with an independently varying radius inclination to be treated as one complex dimension. No simple explanation concerning the particular rules governing the combination of component dimensions can therefore be provided, and more research is necessary to uncover the factors that influence this combination. Instead of considering this variability as an indication that the notion of integrality and separability is a "myth" (cf. Cheng & Pachella, 1984), it may be more informative to attempt to discover the causes of this variability.

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