In the Proceedings of the 1984

Japan Applied Statistical Society Meeting, P.30-41

Basic Concepts of Multidimensional Scaling and Its Applications

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1. Introduction:

Multidimensional scaling (MDS) is a data analysis technique to locate a set of points in a multidimensional space in such a way that points corresponding to similar objects are located close together, while those corresponding to dissimilar objects are located far apart. To take a simple example many roadmaps have a matrix of inter-city distances; Put simply what MDS does is to recover a map based on the inter-city distances. Given a map it is relatively easy to measure a euclidian distance between the cities. However, the reverse operation, that of recovering a map from inter-city distances (or locating the cities in such a way that their mutual distances best agree with a given set of distances) is no easy matter. MDS, roughly speaking, is a method to perform this reverse operation.

Let us look at Figure 1B. This is a matrix of airplane distances between 10 U.S. cities (Kruskal & Wish, 1978). Given a matrix of this sort it would be difficult to find geographic locations of these cities, unless we are very knowledgable about the geography of North America. Most of us know that the 10 cities should be located as in Figure 1A. This is because we already have a fairly accurate internalized map of North America, but it would still be considerably difficult for those who do not know very much about the geography of North America to figure out the relative locations of these cities. A researcher is deemed like those who do not know the geography; the role of MDS is to construct a map like this from a matrix like the one in Figure 1B for those who do not know the "geography" in certain areas. In the remaining time I would like to elaborate this role of MDS through various examples.

2. Morse Code Signals

Objects represented in a map do not have to be intrinsically geometrical. The first example illustrates this point (Figure 2). This is called a confusion matrix (Rothkopf, 1957). (It looks confusing!) Stimuli are 36 Morse Code signals. Signals are presented successively in pairs, and subjects are asked to judge whether the two stimuli are the "same" or "different". The relative frequencies with which row stimuli are judged the "same" as column stimuli are entered in this table. For example, we find that

A (· -)	A (· -)	92%
A (· -)	K (-·-)	22%
A (· -)	T (-)	6%

The most impressive aspect of this table is the overwhelming array of numbers. It is difficult, if not impossible, to find a hidden structure underlying a confusion process between the Morse Code signals by merely inspecting the table.

Figure 3 presents a result of MDS of these data (Shepard, 1963). MDS locates the stimulus points in such a way that more confusable (or more similar) stimuli are located close to each other, while less confusable stimuli are located far apart. The advantage of MDS analysis is rather obvious in this case. If we look at the figure it is readily apparent that:

1. A process mediating the confusions between the Morse Code signals is two dimensional.

2. One is the total number of components in a signal (i.e., the number of dots plus the number of dashes), and the other is the ratio of the number of dots to the number of dashes in a signal (i.e., which is more predominant?)

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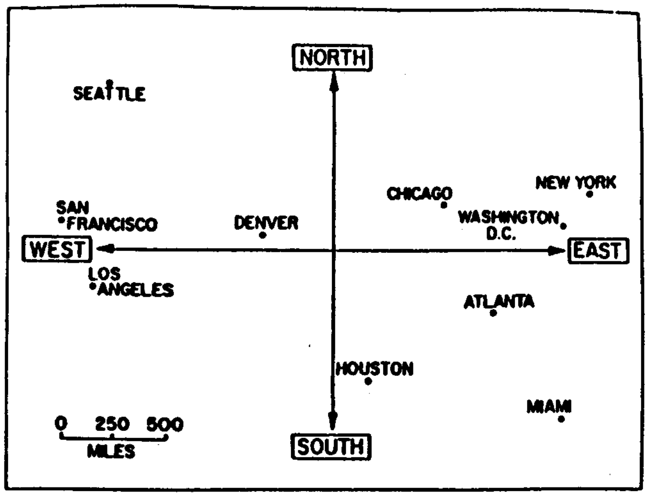
Fig.1 MDS of ten US cities



A) GEOGRAPHIC LOCATIONS OF TEN U.S. CITIES

CITIES	ATLA.	CHIC.	DENV.	HOUS.	L.A.	MIAMI	N.Y.	S.F.	SEAT.	WASH. D.C.
ATLANTA		587	1212	701	1936	604	748	2139	2182	543
CHICAGO	587		920	940	1745	1188	713	1858	1737	597
DENVER	1212	920		879	831	1726	1631	949	1021	1494
HOUSTON	701	940	879		1374	968	1420	1645	1891	1220
LOS ANGELES	1936	1745	831	1374		2339	2451	347	959	2300
MIAMI	604	1188	1726	968	2339		1092	2594	2734	923
NEW YORK	748	713	1631	1420	2451	1092		2571	2408	205
SAN FRANCISCO	2139	1858	949	1645	347	2594	2571		678	2442
SEATTLE	2182	1737	1021	1891	959	2734	2408	678		2329
WASHINGTON D.C.	543	597	1494	1220	2300	923	205	2442	2329	

B) AIRLINE DISTANCES BETWEEN TEN U.S. CITIES

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	1	2	3	4	5	6	7	8	9	0	
A	92	04	06	13	03	14	10	13	46	05	22	03	25	34	06	06	09	35	23	06	37	13	17	12	07	03	02	07	05	05	08	06	05	06	02	03	A
B	05	84	37	31	05	28	17	21	05	19	34	40	06	10	12	22	25	16	18	02	18	34	08	84	30	42	12	17	14	40	32	74	43	17	04	04	B
C	04	38	87	17	04	29	13	07	11	19	24	35	14	03	09	51	34	24	14	06	06	11	14	32	82	38	13	15	31	14	10	30	28	24	18	12	C
D	08	62	17	88	07	23	40	36	09	13	81	56	08	07	09	27	09	45	29	06	17	20	27	40	15	33	03	09	06	11	09	19	08	10	05	06	D
E	06	13	14	06	97	02	04	04	17	01	05	06	04	04	05	01	05	10	07	67	03	03	02	05	06	05	04	03	05	03	05	02	04	02	03	03	E
F	04	51	33	19	02	90	10	29	05	33	16	50	07	06	10	42	12	35	14	02	21	27	25	19	27	13	08	16	47	25	26	24	21	05	05	05	F
G	09	18	27	38	01	14	90	06	05	22	33	16	14	13	82	52	23	21	05	03	15	14	32	21	23	39	15	14	05	10	04	10	17	23	20	11	G
H	03	45	23	25	09	32	08	87	10	10	09	29	05	08	08	14	08	17	37	04	36	59	09	33	14	11	03	09	15	43	70	35	17	04	03	03	H
I	64	07	07	13	10	08	06	12	93	03	05	16	13	30	07	03	05	19	35	16	10	05	08	02	05	07	02	05	08	09	06	08	05	02	04	05	I
J	07	09	38	09	02	24	18	05	04	05	22	31	08	03	21	63	47	11	02	07	09	09	09	22	32	28	67	66	33	15	07	11	28	29	26	23	J
K	05	24	38	73	01	17	25	11	05	27	91	33	10	12	31	14	31	22	02	02	23	17	33	63	16	18	05	09	17	08	08	18	14	13	05	06	K
L	02	69	43	45	10	24	12	26	09	30	27	86	06	02	09	37	36	28	12	05	16	19	20	31	25	59	12	13	17	15	26	29	36	16	07	03	L
M	24	12	05	14	07	17	29	08	08	11	23	08	96	62	11	10	15	20	07	09	13	04	21	09	18	08	05	07	06	06	05	07	11	07	10	04	M
N	31	04	13	30	08	12	10	16	13	03	16	08	59	93	05	09	05	28	12	10	16	04	12	04	06	11	05	02	03	04	04	06	02	02	10	02	N
O	07	07	20	06	05	09	76	07	02	39	26	10	04	08	86	37	35	10	03	04	11	14	25	35	27	27	19	17	07	07	06	18	14	11	20	12	O
P	05	22	33	12	05	36	22	12	03	78	14	46	05	06	21	83	43	23	09	04	12	19	19	19	41	30	34	44	24	11	15	17	24	23	25	13	P
Q	08	20	38	11	04	15	10	05	02	27	23	26	07	06	22	51	91	11	02	03	06	14	12	37	50	63	34	32	17	12	09	27	40	58	37	24	Q
R	13	14	16	23	05	34	26	15	07	12	21	37	14	12	12	29	08	87	16	02	23	23	62	14	12	13	07	10	13	04	07	12	07	09	01	02	R
S	17	24	05	30	11	26	05	59	16	03	13	10	05	17	06	06	03	18	96	09	56	24	12	10	06	07	08	02	02	15	28	09	05	05	05	02	S
T	13	10	01	05	46	03	06	06	14	06	14	07	06	05	06	11	04	04	07	96	08	05	04	02	02	06	05	05	03	03	03	08	07	06	14	06	T
U	14	29	12	32	04	32	11	34	21	07	44	32	11	13	06	20	12	40	51	06	93	57	34	17	09	11	06	06	16	34	10	09	09	07	04	03	U
V	05	17	24	16	09	29	06	39	05	11	26	43	04	01	09	17	10	17	11	06	32	92	17	57	35	10	10	14	28	79	44	36	25	10	01	05	V
W	09	21	30	22	09	36	25	15	04	25	29	18	15	06	26	20	25	61	12	04	19	20	86	22	25	22	10	22	19	16	05	09	11	06	03	07	W
X	07	64	45	19	03	28	11	06	01	35	50	42	10	08	24	32	61	10	12	03	12	17	21	91	48	26	12	20	24	27	16	57	29	16	17	06	X
Y	09	23	62	15	04	26	22	09	01	30	12	14	05	06	14	30	52	05	07	04	06	13	21	44	86	23	26	44	40	15	11	26	22	33	23	16	Y
Z	03	46	45	18	02	22	17	10	07	23	21	51	11	02	15	59	72	14	04	03	09	11	12	36	42	87	16	21	27	09	10	25	66	47	15	15	Z
1	02	05	10	03	03	05	13	04	02	29	05	14	09	07	14	30	28	09	04	02	03	12	14	17	19	22	84	63	13	08	10	08	19	32	57	55	1
2	07	14	22	05	04	20	13	03	25	26	09	14	02	03	17	37	28	06	05	03	06	10	11	17	30	13	62	89	54	20	05	14	20	21	16	11	2
3	03	08	21	05	04	32	06	12	02	23	06	13	05	02	05	37	19	09	07	06	04	16	06	22	25	12	18	64	86	31	23	41	16	17	08	10	3
4	06	19	19	12	06	25	14	16	07	21	13	19	03	03	02	17	29	11	09	03	17	55	08	37	24	03	05	26	44	89	42	44	32	10	03	03	4
5	08	45	15	14	02	45	04	67	07	14	04	41	02	00	04	13	07	09	27	02	14	45	07	45	10	10	14	10	30	69	90	42	24	10	06	05	5
6	07	80	30	17	04	23	04	14	02	11	11	27	06	02	07	16	30	11	14	03	12	30	09	58	38	39	15	14	26	24	17	86	69	14	05	14	6
7	06	33	22	14	05	25	06	04	06	24	13	32	07	06	07	36	39	12	06	02	03	13	09	30	30	50	22	29	18	15	12	61	85	70	20	13	7
8	03	23	40	06	03	15	15	06	02	33	10	14	03	06	14	12	45	02	06	04	06	07	05	24	35	50	42	29	16	16	09	30	60	89	61	26	8
9	03	14	23	03	01	06	14	05	02	30	06	07	16	11	10	31	32	05	06	07	06	03	08	11	21	24	57	39	09	12	04	11	42	56	91	78	9
0	09	03	11	02	05	07	14	04	05	30	08	03	02	03	25	21	29	02	03	04	05	03	02	12	15	20	50	26	09	11	05	22	17	52	81	94	0
	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	1	2	3	4	5	6	7	8	9	0	

↑

Fig.2 Confusion data for 36 Morse code signals

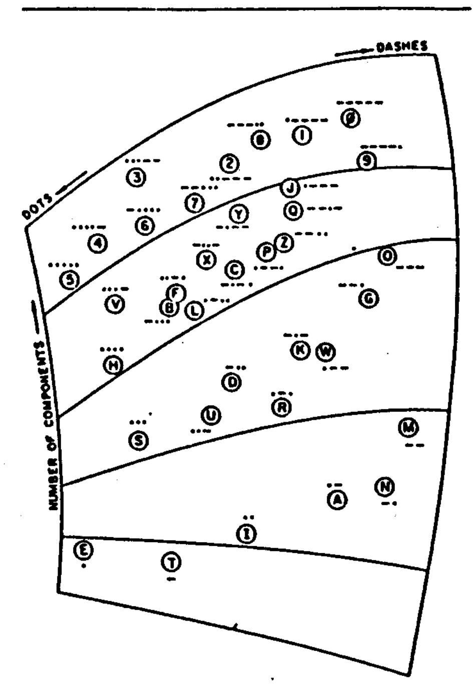
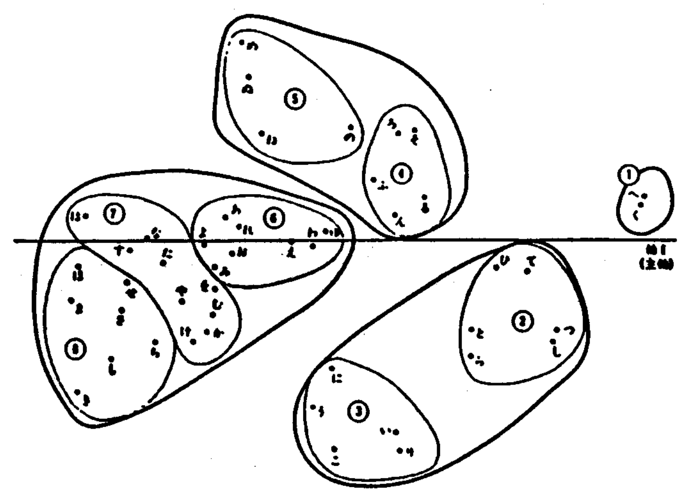


Figure 3 : Shepard's Interpretation of Morse Code Configuration

Fig.4 MDS conf. of Japanese Kana characters →



As can be seen, as we go from the bottom to the top of the figure signals have more components. Stimuli with the same number of components are relatively confusable. Points are more crowded toward the top of the figure, but this merely reflects the fact that a more variety of signals can be composed with more components, and are thus more confusable.

Within the same number of components, symbols on the right have more dashes and fewer dots than those on the left. Stimuli with the same number of dots and the same number of dashes are more confusable, particularly when they are mirror images of each other. Two signals are said to be mirror images of each other when they have exactly the same number of components of each kind, but the components are arranged in exactly reverse orders. In the figure, stimuli which are mirror images of each other are indicated by connected line segments.

3. Japanese Kana Characters

Figure 4 is an MDS configuration of 46 Japanese Kana characters (phonetic symbols). Twenty University of Hawaii students who did not know Japanese were asked to classify these symbols into as many groups as they liked in terms of their similarities in shape (Dunn-Rankin & Leton, 1973). The frequencies with which two symbols are classified into the same groups are taken as similarity measures between these symbols, and MDS is applied to obtain the stimulus configuration.

Unlike the previous example, this configuration does not seem to permit a staightforward dimensional interpretation, though perhaps the horizontal direction roughly represents the complexity in shape (simple (right) vs. complex (left)). The vertical axis is difficult to interpret. However, when cluster analysis, which groups similar objects together, was applied to the same set of data, and the groupings of these stimuli obtained from cluster analysis were superimposed on the stimulus configuration obtained from MDS, it became apparent that another kind of interpretation (not dimensional, but configural) was possible. Our interpretation of six clusters are as follows:

1) angular form
2) curved feature
3) discrete components
4) zigzag feature
5) round feature
6) crossed feature

Of these, the sixth cluster is a major cluster that includes three sub-clusters labeled 6, 7 and 8 in the figure. Note that toward the left end of this cluster characters having a "double cross" are located.

The organizing principle underlying the perceptions of similarities between these symbols seems to be "distinctive features" subsets of the symbols share in common.

4. HAVE words

The data collection method, in which subjects are asked to sort a set of stimuli according to similarity among them, is called the sorting method. It is very popular among social scientists because of its simplicity. We give another example in which the sorting method was used as a data collection method. Stimuli are 29 Have words given in the left margin of Figure 5. Ten university students were asked to classify them into as many groups as they liked in terms of their similarity in meaning. A somewhat specialized MDS method was applied to these data (Takane, 1980), and the result is presented in Figure 5. This configuration permits a straightforward dimensional interpretation.

The horizontal direction distinguishes two future states of the current state of possession. Words like "have", "own" and "belong" are located on the left side of the configuration, while "give", "sell", "lose", etc. are placed on the opposite side. Thus, the horizontal direction contrasts a stable state of possession (left) with a state of possession which is about to change (right). Similarly, the vertical direction distinguishes two states of nonpossession, a stable nonpossession at the top and an unstable state of nonpossession at the bottom. It seems that the vertical direction represents subtle gradients of sureness of change in the state. That is, "lack", "need" and "want" are located at the top, which indicate no prospect of change whatsoever, while "receive", "get", "find", etc. are located at the bottom, which indicate that the change is most probable. Interestingly "beg" is located at about the middle, which indicates that some action has been taken to change the state, but it is not certain if the change will really occur.

5. Color Space for the Pigeon

MDS is not restricted to human subjects. The next example shows this instance (Schneider, 1972). Pigeons are trained to discriminate between colors. A pair of colors are presented as two halves of a circle, as illustrated in the figure below. Pigeons are trained to pack the left lever when two colors are the same, and the right lever when they are different. The relative frequency of incorrect responses for each pair of colors is taken as a measure of similarity

1	Accept
2	Beg
3	Belong
4	Borrow
5	Bring
6	Buy
7	Earn
8	Find
9	Gain
10	Get
11	Get rid of
12	Give
13	Have
14	Hold
15	Keep
16	Lack
17	Lend
18	Lose
19	Need
20	Offer
21	Own
22	Receive
23	Return
24	Save
25	Sell
26	Steal
27	Take
28	Use
29	Want

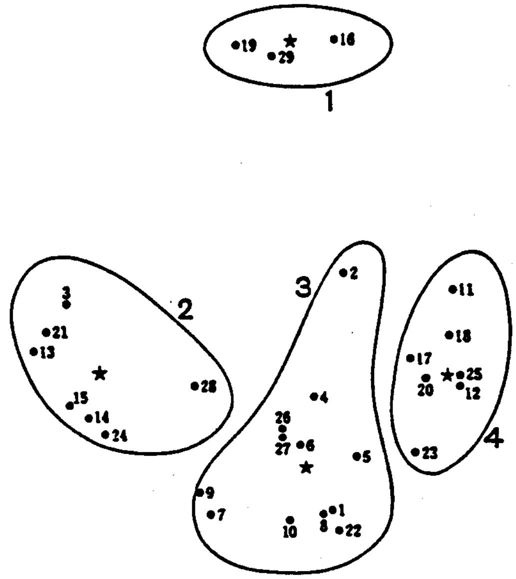


Fig. 5 Plot of stimulus configuration for the HAVE data, and clusters and cluster centroids for subject 6

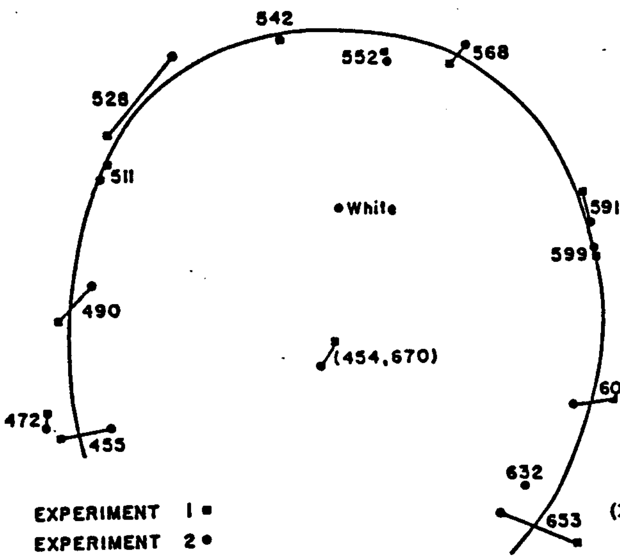


Fig. 6. A two-dimensional color space for the pigeon.

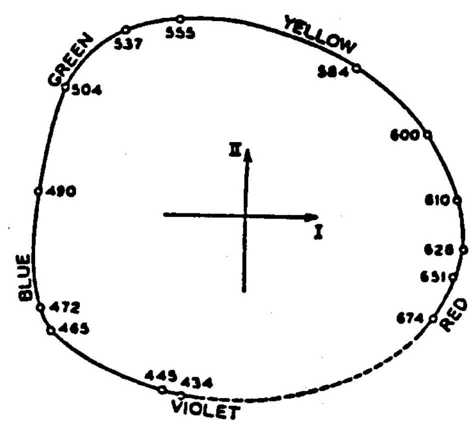
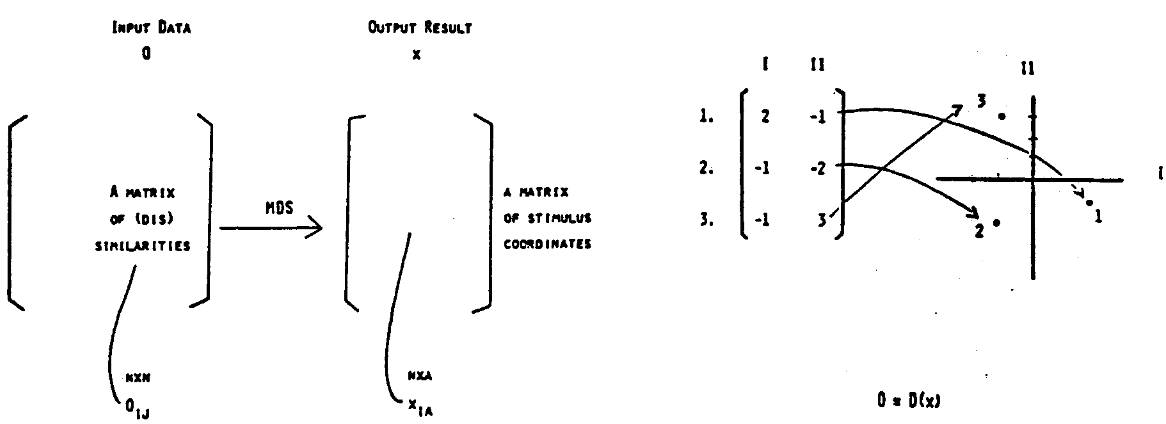


Fig. 7. A two-dimensional color space for humans. Data collected by Ekman (1954), analyzed by Shepard (1964).

Fig. 8 What MDS does.



between the colors.

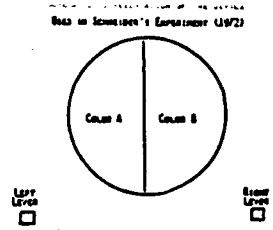


Figure 6 is the stimulus configuration obtained from the confusion data. As we can see, a familiar color wheel is present in pigeon's color space. Figure 7 is the color wheel typically found in human subjects (Ekman, 1964; Shepard, 1962). The two configurations look very much alike with each other.

6. What MDS does

Let us summarize what MDS does at this point (Figure 8). We are given a matrix of (dis)similarities between objects as input data. These data are generally represented by an n by n matrix O whose (i,j) entry o_{ij} is the (dis)similarity between objects i and j, where n is the number of objects. By applying MDS to the data we obtain an n by A matrix X of stimulus configuration, where A is the dimensionality of the multidimensional space in which objects are embedded. The (i,a) element x_{ia} of matrix X represents the coordinate of object i on dimension a. The information contained in X may be further converted into a graphic representation (like those we have already seen) by introducing a Cartesian coordinate system, as indicated in the right portion of the figure. Since the euclidian distance is invariant over the shift of origin and the rotation of axes, we may subsequently remove the coordinate axes (which we have used to locate the stimulus points).

Once X is obtained, then the matrix D(X) of interpoint distances can be calculated (based on the X). MDS determines locations of stimulus-points so that the interpoint distance D(X) in some sense best agree with the observed (dis)similarities O.

7. Helm's Color Data

Various extensions of this basic scheme of MDS is possible; the next example represents one such possibility (Figure 9). We have so far been focusing on an MDS technique which derives a single stimulus configuration given a single matrix of observed (dis)similarities. Suppose we have N ($\geqslant 1$) such matrices, each contributed by a different subject. If no systematic individual differences are suspected, we may analyze them simultaneously and derive a single common stimulus configuration on the basis of the premise that these matrices are mere replications of one another. Alternatively when some individual differences are suspected, we may apply MDS separately to each single matrix. In this case we obtain N stimulus configurations.

The question is whether there is a better way to represent differences among matrices of (dis)similarities than applying MDS separately to these data. The answer is "yes", and a technique is called individual differences MDS (Carroll & Chang, 1970).

Figure 9 helps clarify the kind of analysis performed by the individual differences MDS. Dissimilarity judgments among 10 colors were obtained from 14 subjects (Helm, 1964). The stimulus configuration (the figure on the left) has been obtained by applying the individual differences MDS to these data. We can see the familiar color wheel with:

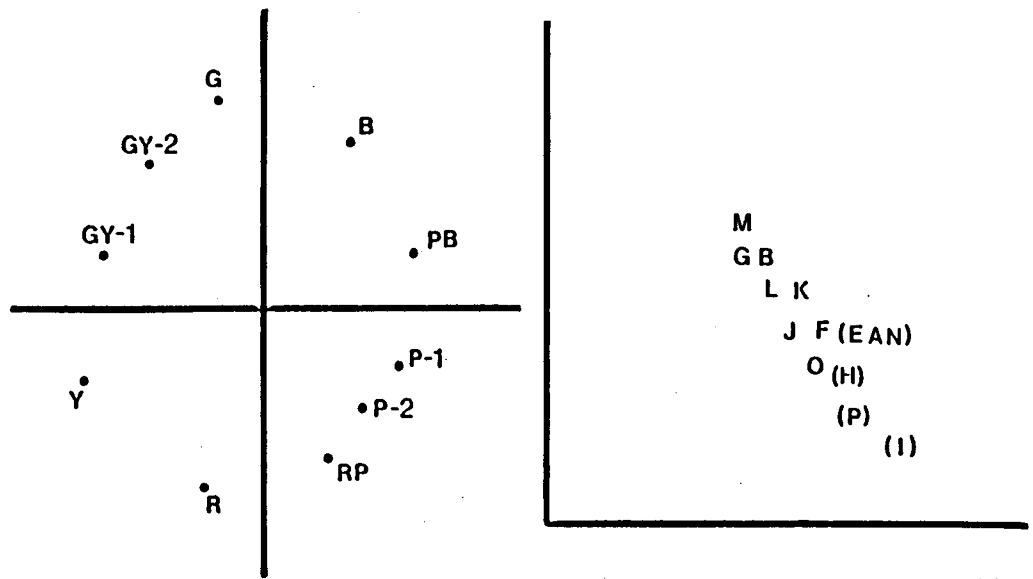
1) Vertical axis representing the contrast between red and green (R-G)

2) Horizontal axis representing the contrast between yellow and blue (Y-B)

The figure on the right represents the weights attached to these two dimensions by different subjects. Subject M, for example, put about equal emphasis on both dimensions, while subject I put excessively heavy emphasis on the horizontal axis. This implies that M's judgments of similarities are based on the two dimensions equally considered, while I's judgments are almost exclusively based on the Y-B dimensions. In fact, subjects with parentheses are color deficient subjects (in G-R), and for them G-R dimension is almost totally missing. Individual differences MDS thus attempts to describe individual differences in (dis)similarity judgments by differential weights attached to dimensions by different individuals (Figure 10).

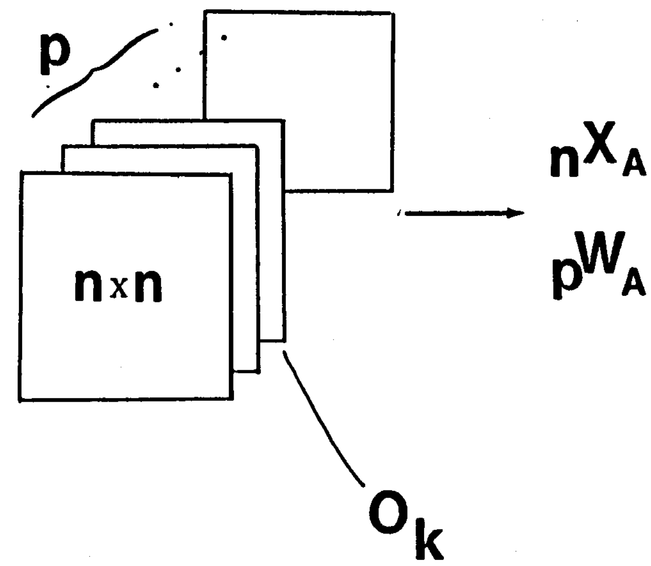
Shepard & Cooper (1975) have done an interesting follow-up study on the color space of color deficient subjects. They found that the color space derived from the color deficient subject, based on color names (rather than actual colors) was very much like a color wheel similar to that usually obtained from color normal subjects. This means that, although the color deficient subjects can never see actual colors, they are aware, to some extent, of their conceptual relationship.

Fig.9 INDIVIDUAL DIFFERENCES ANALYSIS OF COLOR PERCEPTIONS



(From Carroll & Chang, 1970; data from Helm, 1964)

Fig.10 What ID MDS does



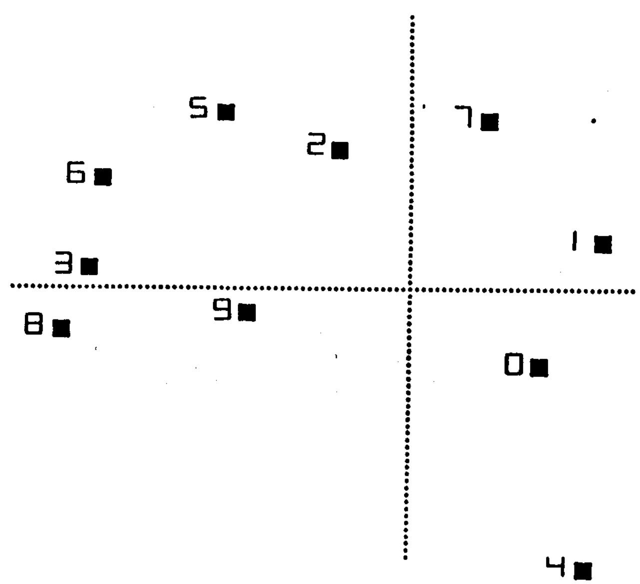


Fig.11

DERIVED 2 DIMENSIONAL CONFIGURATION FOR SUBJECT 1 FOR DIGITS

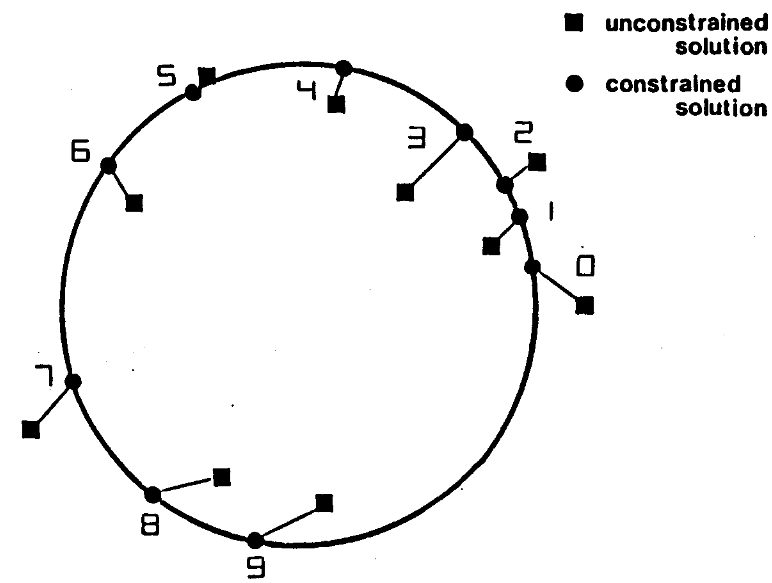


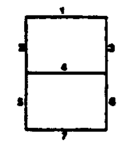
Fig.12

DERIVED 2 DIMENSIONAL STIMULUS CONFIGURATION FOR SUBJECT 2 FOR DIGITS

8. Segmented Numerals

Of cource, not all types of individual differences can be represented by differential weighting of dimensions. Figures 11 and 12 present results of (simple) MDS of segmented numerals from two subjects (Sergent & Takane, in preparation.) The stimuli are similar to those used for displaying time in digital watches. That is, they are constructed by picking appropriate segments from seven basic constituents indicated in the following figure.

Segments used in the digits



For example, digit one is composed of segments 3 and 6, two is composed of segments 1, 3, 4, 5 and 7, and so on. Those stimuli were presented in pairs. Reaction times that the subjects took to judge if two stimuli presented were the "same" or "different" were used as input data (Takane & Sergent, 1983).

The two configurations seem to be completely unrelated. For subject 1 (Figure 11) the stimuli are organized according to the constituent segments. The horizontal axis divides (roughly at the vertical dotted line) numerals all having two top (or three) horizontal line segments (on the left) from those without them (on the right). The vertical axis divides (roughly at the horizontal dotted line) numerals with three vertical segments 2, 3 & 6 (bottom) from those lacking at least one of them. For subject 2 the configuration circles around, starting from 0 and ending with 9. This type of configuration is often obtained when a two dimensional solution is obtained for one dimensional stimuli due perhaps to the ceiling effect of the similarity measure. In any case this subject seems to perceive the stimuli as numbers (rather than mere collections of particular segments), since it took him more time to discriminate two numbers which are similar in numerical value.

The way the same set of stimuli are perceived is entirely different for these two subjects. Clearly this type of difference cannot be captured by the individual differences MDS described earlier.

9. MDS of line drawing of faces

For multiple sets of data to be adequately described by the individual differences MDS they must exhibit certain patterns when analyzed separately by simple MDS. The next example shows such patterns.

The stimuli are eight faces (Figure 13) constructed by factorially combining two levels each of three features (Table 1). As in the previous example reaction times were taken and used as input data. Figures 14 and 15 display stimulus configurations derived from two subjects by applying simple MDS separately. Four dimensions obtained agree between the two subjects. The four dimensions are interpreted, in the order of salience, as follows:

1. Hair
2. Jaw
3. Eye
4. Sex consistency.

The first three dimensions correspond with the three defining features of the stimuli. The last one, a bit difficult to interpret, is the additional dimension used by the subjects in performing the RT task. On the fourth dimension stimuli most consistent with typical sex profiles (short hair, angular jaw and dark eyes for males, and long hair, round jaw and light eyes for females) are located at the top, while remaining stimuli are located downward according to their degree of inconsistency with the sex profiles.

The most striking thing is that the two configurations are remarkably similar, in fact, almost identical except for the dimensional saliency. The individual differences MDS discussed earlier is most appropriate to describe this kind of differences in the dimensional saliency.

10. Facial Expressions Data

We give a few more examples of the individual differences MDS. Figure 16 shows a set of stimuli employed. They are constructed by factorially combining two of the most important determinants of facial expressions, namely the curvature of eyes and the curvature of lips (Inukai, 1981). Figure 18 shows the common stimulus configuration obtained by applying individual differences MDS. We see that the two defining properties of stimuli (eyes and lips) are also present in this subjective space. That is, subjects' judgments of similarities are organized around the two physical attributes with the vertical axis roughly corresponding with the

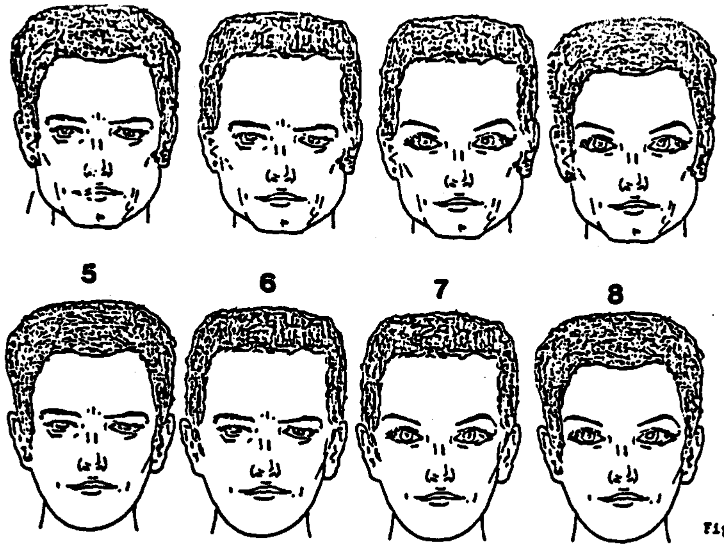


Fig.13 Stimuli used
in Takane & Sergent (1983)

Table 1. The stimuli and their features.

Stimulus	Features: hair (H)	eye (E)	jaw (J)
1	Long	Dark	Angular
2	Short	Dark	Angular
3	Short	Light	Angular
4	Long	Light	Angular
5	Long	Dark	Round
6	Short	Dark	Round
7	Short	Light	Round
8	Long	Light	Round

Figure 14 Derived stimulus configuration for subject 1.

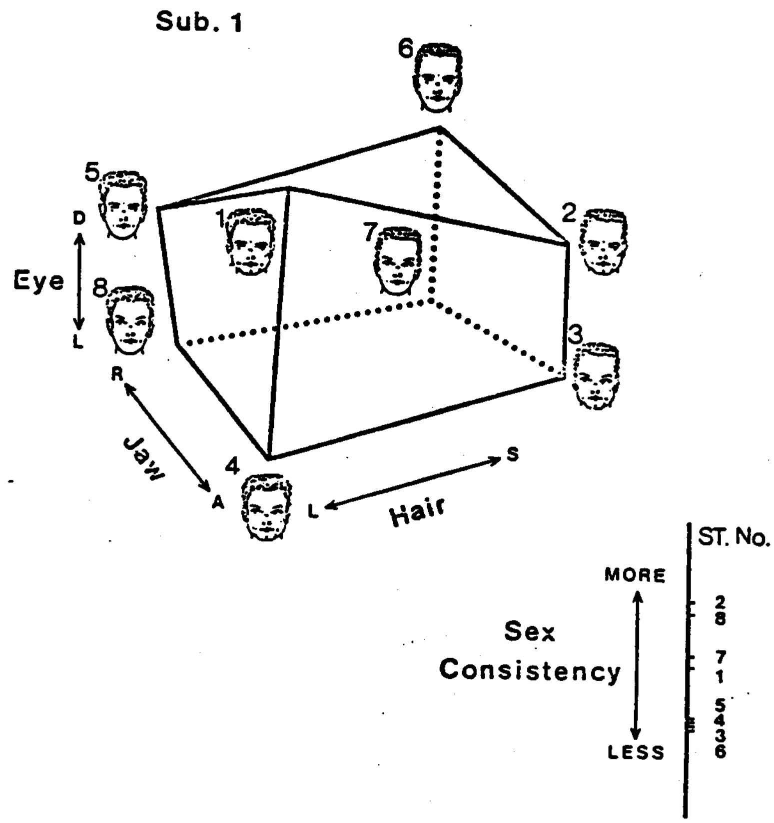


Figure 15 Derived stimulus configuration for subject 2.

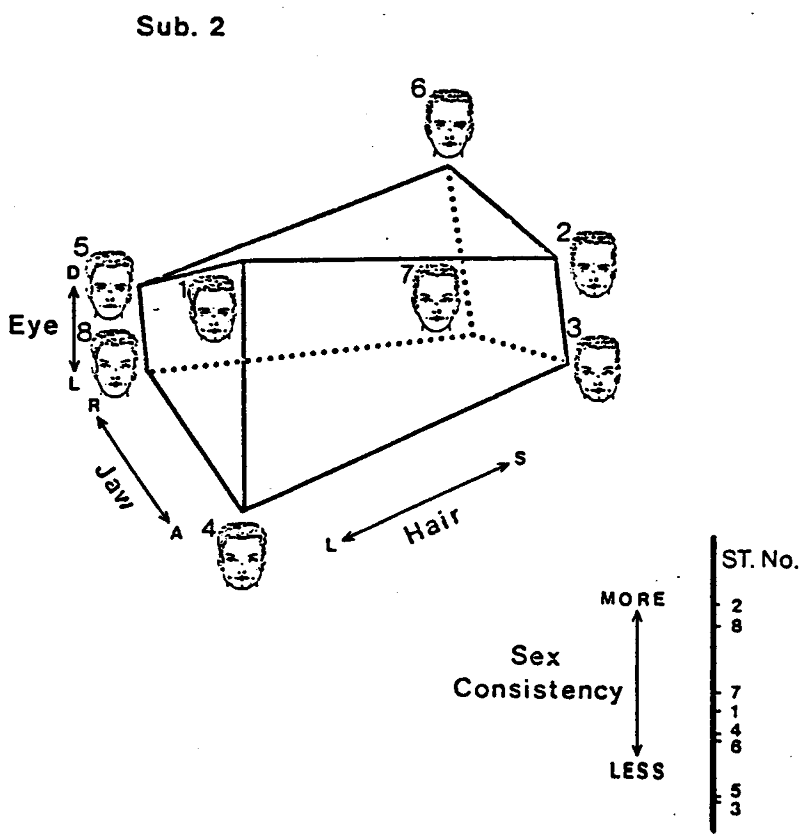


Fig.16

STIMULI USED IN THE FACIAL EXPRESSION EXPERIMENTS

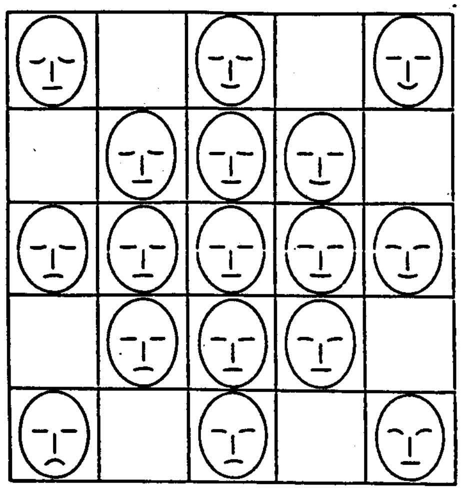


Fig.17

INDIVIDUAL DIFFERENCES WEIGHTS ATTACHED TO THE EYES AND LIPS DIMENSIONS

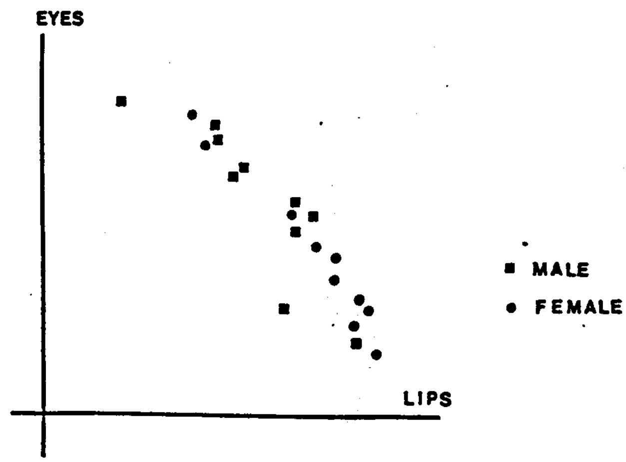
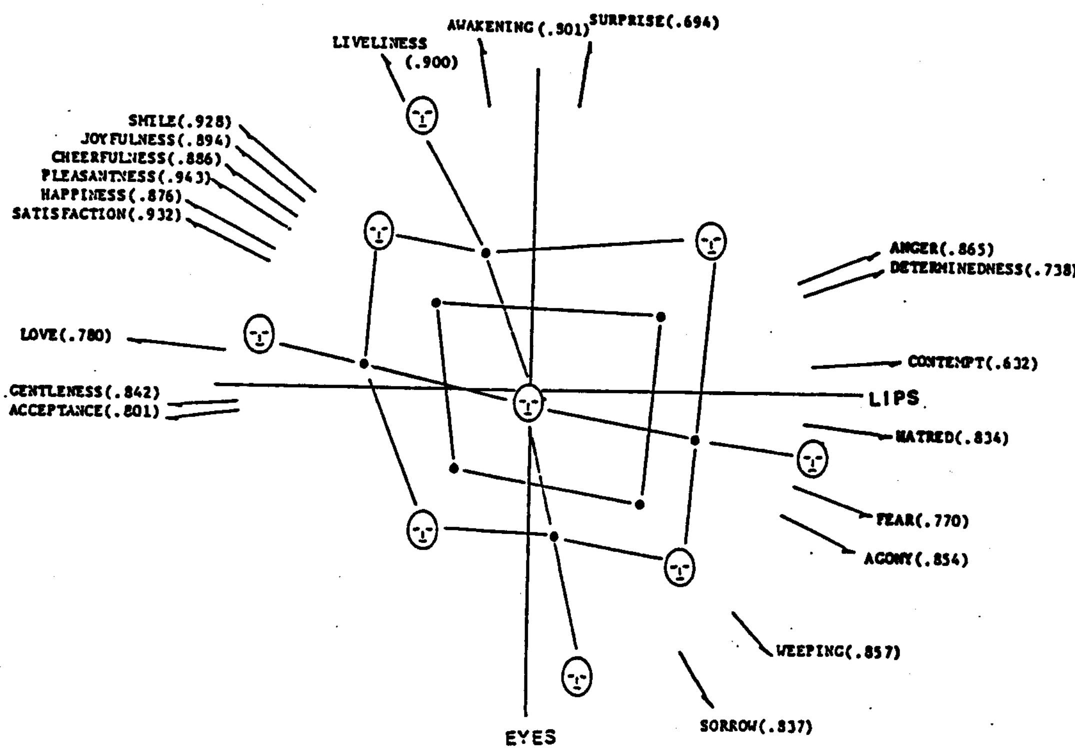


Fig.18 SPATIAL REPRESENTATION OF FACIAL EXPRESSIONS AMD EMOTIONAL ORIENTATIONS



eyes dimension and the horizontal axis with the lips dimension.

In addition to the dissimilarity data, the experimenters also obtained rating judgments on the same set of stimuli regarding the extent to which they express certain emotional dispositions. Directions (designated by arrows in the figure) indicate the directions with which certain emotional dispositions are most highly correlated. Members in parentheses are multiple correlation coefficients. For example, "smile" is in quadrant 2, and that is the direction in which this emotional state is most clearly represented. Similarly we may characterize other emotional states as:

happiness - moderate concave eyes, moderate convex lips
alertness - high concave eyes, straight lips
anger - straight eyes, high concave lips
weeping - moderate convex eyes, moderate concave lips
sleeping - ~~high~~ convex eyes, straight lips.
[illegible]

Interestingly enough there seem to be fairly clear sex differences in the evaluation of these two dimensions. Figure 17 shows individual differences weights. Squares represent weights for male subjects and circular dots those for female subjects. Generally speaking, the female subjects put more emphasis on the lips dimension than the male subjects.

11. Body-Parts Data

The next example is an application of the individual differences MDS to describe a developmental change in concepts (Takane, et. al., 1977). Dissimilarity judgments were obtained between various body-part names from a group of adult subjects and a group of 6-year-old children. Figure 19 represents a common stimulus configuration. The solution is three-dimensional, where dimensions are interpreted as follows:

Dimension

1. Contrasts between face terms and limbs (both lower and upper) terms with "body" in between.

2. Contrasts between upper and lower limbs with "body" and all face terms in the middle.

3. Represents whole - part hierarchy. (The term "body" is right in front, and finer and finer parts of the body are located in the back.)

An interesting thing is that there is a fairly systematic developmental difference in the weighting of these dimensions (Figure 20). The next figure represents individual differences weight. White cubes indicate 6-year-old children's weights, while black cubes represent those for adults. As we can see, 6-year-old children tend to put more emphasis on dimension 2 than dimension 1 or 3, while adults are more heterogeneous among themselves. They split into two groups, one placing more emphasis on dimension 1 and the other on dimension 3, but no adults put most emphasis on dimension 2. The distinction between upper and lower limbs is very important for young children. As they grow older they realize certain parallelism between upper and lower limbs.

12. Family Composition Preference Data

MDS is not restricted to the usual (dis)similarity data. The last example represents preference toward family compositions; i.e., the number of boys and of girls the subject would like to have. Stimuli are various family compositions constructed by factorially combining different levels of the number of boys (0-3) and the number of girls (0-3). Preference orders are obtained for the 16 stimuli from 82 Belgian students (Figure 21). In the figure more preferred compositions are indicated by smaller numbers. For example, for the first subject (1,1) is the most preferred composition, and (2,1) is the next most preferred composition and so on, where (m,f) indicates the composition in which the number of boys is m and the number of girls is f. One way to analyze these data is a simple extension of MDS, in which it is assumed that:

1) Each subject has an ideal point in a multidimensional space (corresponding to his or her ideal stimulus).

2) The closer a stimulus point is to one's ideal point, the more preferred that stimulus is by that particular subject.

MDS in this case locates subjects' ideal points as well as stimulus points in a joint space in such a way that more preferred stimuli are located close to ideal points, while less preferred stimuli are located far apart from the ideal. This analysis is called unfolding analysis (Figure 23).

Figure 22 shows the result of the unfolding analysis of the preference data given in Figure 21 (Heiser, 1981). In the figure stimulus points are indicated by a pair of numbers (e.g., 1,3), while subjects' ideal points are indicated by x's. We see that there is a strong tendency to prefer large families among the Belgian students. We can also observe a slight boy bias (preference for having more boys). It is also evident that the difference between two small families (e.g., (1,0) and (1,1)) is much larger than the difference between two large families (e.g., (2,2) and (3,3)). Adding one girl is important when there is no girl, but it is not very significant when there are two girls already in the family. Again, the important point is that these observations, so obvious in Figure 22, can be hardly made by merely inspecting the data table.

The unfolding model has had considerable impacts on marketing research (e.g., developing ideal products). Figure 23 illustrates schematically what the unfolding analysis does.

13. MDS texts in Japanese

To know more about MDS the interested reader is referred to Kruskal & Wish (1978; translated by Takane and published by Asakura Shoten, 1980) for applications of MDS. Takane (1980) mainly contains theoretical and methodological work on MDS up to late 1970's.

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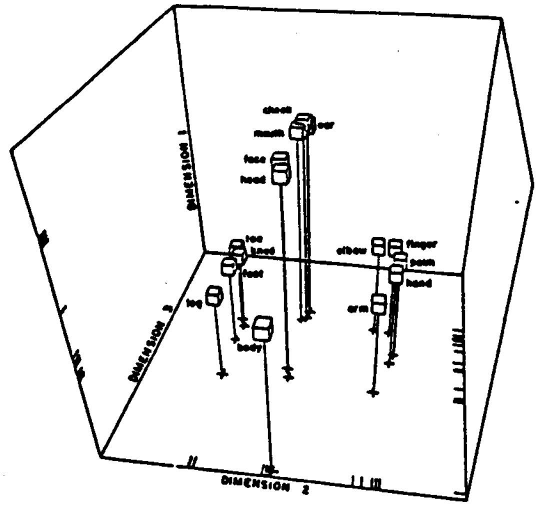

Figure 19
Jacobowitz Body parts data: Three-dimensional stimulus space.

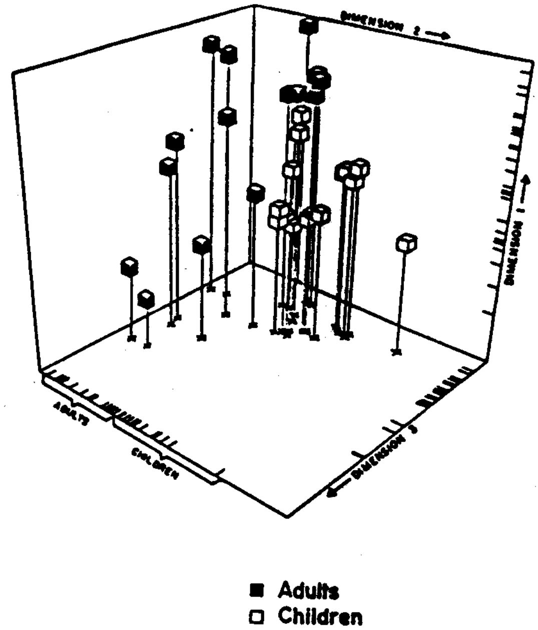

Figure 20
Jacobowitz Body parts data: Three-dimensional weight space

Fig.21

DELBEKE'S DATA ON THE PREFERENCE OF FAMILY COMPOSITION

NO	STIMULI 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	16	6	8	4	7	1	2	12	5	3	10	13	9	11	14	15
2	16	7	4	5	15	1	3	8	10	9	2	6	14	13	11	12
3	16	14	9	7	15	6	4	2	10	5	1	11	8	3	12	13
4	16	15	13	12	14	8	3	5	11	2	1	7	10	4	6	9
5	16	12	10	13	14	8	2	7	11	6	5	1	15	9	3	4
6	16	7	5	6	15	11	1	2	13	10	4	3	14	9	12	8
7	16	12	11	6	15	5	2	3	14	4	1	8	13	7	9	10
8	16	8	6	5	14	9	4	2	15	11	3	1	13	12	7	10
9	14	15	10	12	16	4	2	5	11	3	1	7	13	6	8	9
10	15	16	13	9	14	10	7	8	11	5	4	2	12	6	1	3
11	16	13	4	3	15	1	2	9	11	8	5	7	12	14	10	6
12	16	14	6	3	15	5	2	4	12	8	1	7	13	9	11	10
13	16	14	9	8	15	4	1	11	10	2	5	3	13	12	6	7
14	16	14	10	11	15	9	5	6	13	8	4	2	12	7	3	
15	15	14	12	16	13	11	9	10	8	7	5	6	2	3	1	
16	16	6	2	7	8	1	4	10	3	5	9	11	13	14		
17	16	15	13	10	14	9	8	7	12	5	3	6	11			
18	16	8	5	2	15	3	1	10	9	4	6	7	14			
19	16	14	12	13	15	10	7	6	11	8	1	3				
20	15	12	10	14	11	6	2	5	9	1	3	8				
21	16	15	10	11	13	9	8	5	12	7	1					
22	16	14	11	10	15	9	5	6	13	7						
23	16	11	6	12	13	4	1	5	14	2						
24	16	15	13	12	14	7	8	10	4							
25	16	14	10	9	15	8	6	5	12							
26	16	12	11	10	15	5	6	8								
27	14	15	11	9	16	8	1	5								
28	16	13	12	14	10	1	4									
29	16	14	8	7	15	10										
30	1	2	5	7	4	3										
31	14	15	10	11	16											
32	16	14	11	9	15											
33	16	4	3	8												
34	15	7	6													
35	16	7	6													
36	16	2														
37	16	7														
38	16															
39	16															
40																
41																
42																

Construction of the 16 stimuli

St.No.	m	f
1	0	0
2	1	0
3	2	0
4	3	0
5	0	1
6	1	1
7	2	1
8	3	1
9	0	2
10	1	2
11	2	2
12	3	2
13	0	3
14	1	3
15	2	3
16	3	3

m: No. of boys
f: No. of girls

Fig.22 Joint representation of the stimulus and the ideal points

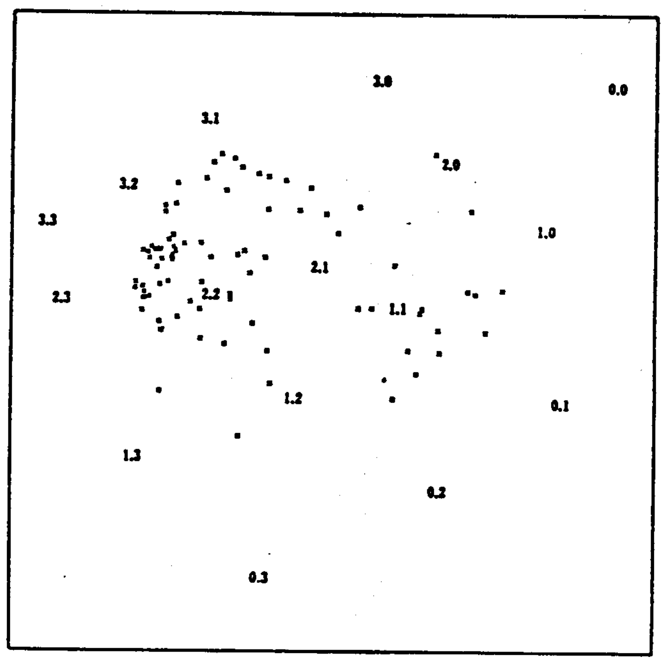

Fig.23 What unfolding analysis does